Computability and Computational Complexity, A.Y. 2025–2026

Written test

Tuesday, July 23, 2026

Exercise 1

For each of the following properties of Turing machines, show whether they are recursive or not. Invoke Rice's Theorem when possible; if it doesn't apply, explain why.

$$\begin{aligned}
\mathcal{P}_1 &= \{\mathcal{M} | \mathcal{M} \text{ halts in at most 100 steps on the empty tape}\};\\
\mathcal{P}_2 &= \{\mathcal{M} | \mathcal{M} \text{ halts in at most 100 steps on every input}\};\\
\mathcal{P}_3 &= \{\mathcal{M} | \mathcal{M} \text{ halts on every input}\};\\
\mathcal{P}_4 &= \{\mathcal{M} | L(\mathcal{M}) \text{ only contains strings of even length}\}.
\end{aligned}$$

Exercise 2

An astronomer has a list of 50 stars that must be photographed by an automated telescope during one night. The telescope starts from its initial resting position and must subsequently point to each of the 50 stars. The time for moving the telescope from one star to the next is proportional to their angular distance. After pointing (and shooting) to all the stars in the list, the telescope must move back to its resting position.

In order to minimize the expensive telescope time, the astronomer writes a program that, given the list of star coordinates, computes the optimal photographing sequence, i.e., the sequence of telescope movements starting and ending at the resting position, covering all 50 stars, and taking the shortest possible time. The program does not directly drive the telescope, and can be run at any moment during the day: on the 50-star list it takes 10 minutes and outputs the optimal sequence. In the night, the sequence is fed to the actual telescope driver and is completed in 2 hours of telescope time.

A few days later, the astronomer has a new list of twice as many (100) stars to be photographed on the next night. In the morning they fire up their program, but it keeps crunching numbers for the whole day until the astronomer, out of patience, kills it.

2.1) Why did this happen? Identify a known hard problem and show by reduction that this behavior is, indeed, expected.

2.2) Using the existing program with a reasonably short runtime, is it still possible for the astronomer to plan a (possibly suboptimal) sequence that still takes all 100 required photographs in one night?

Hint — *For 2.2, you might want to consider a* divide et impera *strategy.*
For simplicity, assume that all required stars are visible throughout the night.

Exercise 3

3.1) Define the time classes **P**, **NP**, and **EXP**.

3.2) Prove that **P** $\subseteq$ **NP** $\subseteq$ **EXP**.