Computability and Computational Complexity, A.Y. 2025–2026

Written test

Wednesday January 7, 2026

Exercise 1

1.1) Is the HALT language **NP**-hard?

1.2) Is HALT **NP**-complete?

Hint — *Motivate your answer starting from the relevant definitions. While this question has a definitely correct answer, you are allowed to express well-motivated doubts!*

Exercise 2

An instance (k, G) of the CLIQUE language (where k is the required clique size and G is an undirected graph) is encoded with alphabet $\Sigma = \{\text{␣}, \texttt{0}, \texttt{1}, \texttt{+}\}$ as follows:

- the binary encoding of k,
- followed by a blank cell,
- followed by the adjacency matrix of G, where every row is represented as a sequence of `0`'s and `1`'s, and rows are separated by a + symbol.

2.1) Does the instance encoded by the following string belong to the CLIQUE language or not?

`11␣0101+1010+0101+1010`

2.2) Write a Turing machine that performs a reduction from the INDEPENDENT SET to the CLIQUE language, using the same encoding for the instances (k, G) of both languages. What is the time complexity of the reduction?

Exercise 3

For each of the following properties of Turing machines $\mathcal{M}$, prove whether it's computable or not. Invoke Rice's theorem whenever it applies.

$$
\begin{aligned}
\mathcal{P}_1 &= \{\mathcal{M} : \mathcal{M} \text{ accepts at least one input string}\} \\
\mathcal{P}_2 &= \{\mathcal{M} : \mathcal{M} \text{ accepts all input strings}\} \\
\mathcal{P}_3 &= \{\mathcal{M} : \mathcal{M} \text{ computes the Kolmogorov complexity of the input string}\} \\
\mathcal{P}_4 &= \{\mathcal{M} : \mathcal{M} \text{ halts on all input strings of length} \leq 10\} \\
\mathcal{P}_5 &= \{\mathcal{M} : \mathcal{M} \text{ halts in no more than 10 steps on all input strings}\}.
\end{aligned}
$$