Computability and Computational Complexity, A.Y. 2024–2025

Written test

Monday, July 21, 2025

Exercise 1

1.1) Given a finite alphabet and using deterministic Turing machines as the underlying computational model, write the definitions of *Recursive* (R) and *Recursively Enumerable* (RE) language.

1.2) Prove that the definition of recursive language does not change if we use non-deterministic TMs (i.e., every language that is recursive wrt DTMs is also recursive wrt NDTMs and vice versa).

Exercise 2

In order to schedule a (very short) exam session, a University department must distribute n exams among $m = 3$ time slots. In general n can be very large, therefore many exams must take place at the same time. To try to avoid conflicts, all students are asked to register to exams beforehand; two exams are *conflicting* if at least one student is registered to both. The department will try to assign conflicting exams to different time slots, so that no student has to sit through two exams at the same time. If such a schedule exists, we call it *non-conflicting*.

2.1) Prove that the problem of deciding the existence of a non-conflicting schedule is in **NP**. In particular, clarify what is the input and what is its size with respect to n (remember that m is fixed to 3).

2.2) Prove that, as the number n of exams grows, the university policy is not scalable, in the sense that there is no known algorithm that can determine the existence of a non-conflicting exam schedule in polynomial time with respect to n (again, remember that $m = 3$).

2.3) Provide polynomial algorithms to decide the problem when $m = 1$, $m = n - 1$ and $m \geq n$. Is the problem still polynomial wrt the number n of exams when $m = 2$?

Hint — *As usual, here is a list of known **NP**-complete problems for reference: SATISFIABILITY, 3-SATISFIABILITY, CLIQUE, INDEPENDENT SET, INTEGER LINEAR PROGRAMMING, VERTEX COVER, 3-VERTEX COLORING, SUBSET SUM, KNAPSACK, HAMILTONIAN PATH, DIRECTED HAMILTONIAN CYCLE, HAMILTONIAN CYCLE, TRAVELING SALESMAN PROBLEM.*

Exercise 3

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ be a three-symbol alphabet. Consider the language $L \subset \Sigma^*$ of strings where the three symbols have the same number of occurrences. For example:

$\mathtt{abbcaccba} \in L$

$\mathtt{abc} \in L$

$\mathtt{abacab} \notin L$

$\varepsilon \in L$

$\mathtt{aabb} \notin L$

$\mathtt{aaabbbccc} \in L$

3.1) Prove that $L \in \mathbf{P}$.

3.2) Prove that $L \in \mathbf{L}$.