Computability and Computational Complexity, A.Y. 2023–2024

Written test

Friday, July 19, 2024

Exercise 1

1.1) Define the **PSPACE** complexity class.

1.2) Prove **P** $\subseteq$ **PSPACE**.

1.3) Prove **NP** $\subseteq$ **PSPACE**.

Exercise 2

The FACTORING decision problem is the following: given a pair of numbers $(n, k) \in \mathbb{N}^2$, does n have a prime factor larger than or equal to k?
For instance, $(28, 5) \in$ FACTORING because 28 is divisible by 7, which is a prime number larger than 5.
On the other hand, $(27, 5) \notin$ FACTORING because the only prime that divides 27 is 3, which is less than 5.

2.1) Prove that FACTORING $\in$ **PSPACE**.

Hint — *This can be achieved in two ways.*
*You can prove it directly, by providing a simple algorithm that scans all prime numbers larger than k and checks if any of them is a divisor of n, and showing that this algorithm is **PSPACE**.*
*Or you can prove it indirectly by showing that FACTORING belongs to a more convenient complexity class that is a subset of **PSPACE** (e.g., **NP**).*
***Bonus points** if you can give both proofs.*

Exercise 3

3.1) Define Radó's n-state, 2-symbol Busy Beaver.

3.2) On July 2nd, 2024, the 5-state, 2-symbol Busy Beaver has been announced. It halts after $S(5) = 47,176,870$ steps leaving $\Sigma(5) = 4,098$ 1's on the tape.
Being aware of this fact, propose a simple algorithm that decides the empty-tape Halting Problem (the version HALT_ε for machines starting on an empty tape) for all Turing Machines with two symbols and at most 5 states.