Computability and Computational Complexity, A.Y. 2023–2024

Written test

Monday, June 17, 2024

Exercise 1

Remember that by "vertex coloring" of an undirected graph we mean the task of assigning an element from a finite set of labels ("colors") to each vertex of the graph so that no two connected vertices have the same color; we say that a graph can be k-colored if the task can be successfully carried out with no more than k colors.

For each of the following coloring-related languages, determine whether they are in **P**, **NP** and/or **coNP** and provide a short motivation for your answers:

1. graphs that can be 2-colored;
2. graphs that cannot be 2-colored;
3. graphs that can be 3-colored;
4. graphs that cannot be 3-colored;
5. graphs $G = (V, E)$ that cannot be k-colored for any $k < |V|$.

Acceptable motivations can be formulated as short descriptions of algorithms (verbal or any kind of pseudocode), set inclusions (e.g., "I already proved that $L \in A$, but we know that $A \subset B$, therefore $L \in B$"), reductions to some **NP**-complete version of SATISFIABILITY (e.g., 3-SAT).

Exercise 2

Choose **one** of the following topics and develop it along the suggested lines.

Topic 1: **Kolmogorov complexity**

1. Given a suitable alphabet Σ and a universal Turing machine (UTM) $\mathcal{U}$, define the Kolmogorov complexity $K_{\mathcal{U}}(s)$ of a string $s \in \Sigma^*$.
2. Show that, given two UTMs $\mathcal{U}$ and $\mathcal{V}$, the difference between the Kolmogorov complexities $K_{\mathcal{U}}(s)$ and $K_{\mathcal{V}}(s)$ of the same string s is bounded by a constant that only depends on $\mathcal{U}$ and $\mathcal{V}$ and not on s.
3. Outline the proof that $K_{\mathcal{U}}$ is uncomputable.

Topic 2: **Post Correspondence Problem**

1. Define the Post Correspondence Problem (PCP).
2. Given the "Restricted" version of the Post Correspondence Problem (RPCP) where the initial pair is fixed (i.e., pre-set as part of the instance), show how an instance of RPCP can be reduced to an equivalent instance of PCP.
3. Outline the proof that PCP is uncomputable.