Computability and Computational Complexity, A.Y. 2023–2024

Written test

Monday, January 8, 2024

Exercise 1

Let L be a **finite, non-empty** language on the alphabet $\Sigma = \{0, 1\}$.
For each of the following propositions say whether it is true or false, and briefly motivate your answer.

1. L is computable.
2. The property "$\mathcal{M}$ decides L," where $\mathcal{M}$ is a deterministic Turing machine, is recursive.
3. The property "the string representing, in binary notation, the number of steps of $\mathcal{M}(\varepsilon)$ before halting belongs to L," where $\mathcal{M}$ is a deterministic Turing machine, is recursive.
4. The property "the string representing, in binary notation, the number of states of $\mathcal{M}$ belongs to L," where $\mathcal{M}$ is a deterministic Turing machine, is recursive.

Exercise 2

Let L be a **finite, non-empty** language on the alphabet $\Sigma = \{0, 1\}$.
For each of the following propositions say if it is true, false or (to the best of our knowledge) unknown, and briefly motivate your answer.

1. $L \in$ **P**.
2. $L \in$ **L**.
3. L is polynomial-time reducible to 3SAT.
4. 3SAT is polynomial-time reducible to L.

Exercise 3

Define the probabilistic time complexity class **RP** and prove the following inclusions:

$$\mathbf{P} \subseteq \mathbf{RP}, \qquad \mathbf{RP} \subseteq \mathbf{NP}, \qquad \mathbf{RP} \subseteq \mathbf{BPP}.$$