Computability and Computational Complexity, A.Y. 2019–2020

Written test

Wednesday, September 2, 2020

Consider the following language on the two-symbol alphabet $\{\mathtt{0}, \mathtt{1}\}$:

$$L = \{\mathtt{0}^n\mathtt{1}^m \,|\, n, m \in \mathbb{N} \wedge n > m\}.$$

In plain terms, a string is in L if and only if it starts with a sequence of 0's followed by a (possibly empty) sequence of 1's and nothing else, with strictly more 0's than 1's.
Some examples:

$$\begin{array}{ccc} \mathtt{00011} \in L & \mathtt{00111} \notin L & \mathtt{0} \in L \\ \mathtt{1} \notin L & \mathtt{10} \notin L & \mathtt{11000} \notin L \\ \mathtt{0110100} \notin L & \mathtt{0000} \in L & \mathtt{0011} \notin L \\ & \varepsilon \notin L. & \end{array}$$

Exercise 1

1.1) Write down a one-tape deterministic Turing Machine $\mathcal{M}$ on the three-symbol alphabet $\{\mathtt{0}, \mathtt{1}, \text{␣}\}$ that, given an input string $s \in \{\mathtt{0}, \mathtt{1}\}^*$, decides $s \in L$.
You may assume that the input string s is surrounded by infinite blank cells ␣ in both directions, and that the initial current position is the leftmost symbol of s.

1.2) What is the time complexity of your machine $\mathcal{M}$?
More precisely: if n is the input size, what is the smallest exponent k such that $\mathcal{M} \in \text{DTIME}(n^k)$? Explain briefly.

Exercise 2

Prove that the language L belongs to the complexity class **L**.

Exercise 3

Is it always possible for an instructor to correctly evaluate a student's answer to 1.1? Explain.