Computability and Computational Complexity, A.Y. 2018–2019

First written test

Mauro Brunato

Thursday, January 10, 2019

Contents

Question sheet 1

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 2

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine always writes the "␣" symbol on the tape.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 3

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

> undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 4

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) ...the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 5

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 6

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine always writes the "␣" symbol on the tape.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 7

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "$\textvisiblespace$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 8

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine can only move left.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 9

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 10

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine always writes the "␣" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 11

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) ...the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 12

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that...

1.1) ...the machine can only move left.

1.2) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "$\llcorner\!\lrcorner$" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 13

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "$\textvisiblespace$" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 14

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine always writes the "␣" symbol on the tape.

1.2) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 15

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

1.2) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 16

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that...

1.1) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "$\llcorner\!\lrcorner$" at least once).

1.2) ...the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 17

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 18

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine always writes the "␣" symbol on the tape.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 19

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) ...the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 20

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 21

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 22

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine always writes the "␣" symbol on the tape.

1.2) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 23

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 24

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 25

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 26

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine always writes the "␣" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 27

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) ...the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 28

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 29

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\sqcup, 1\}$, where "$\sqcup$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 30

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine always writes the "␣" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 31

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 32

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 33

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 34

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine always writes the "␣" symbol on the tape.

1.2) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 35

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

1.2) ...the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 36

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where “$\textvisiblespace$” is the default symbol, such that…

1.1) …the machine can only move left.

1.2) …the machine never writes “1” for 5 consecutive times (i.e., after writing “1” for 4 consecutive steps, it will write “$\textvisiblespace$” at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that “1000” is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words “less than” with “exactly” in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words “less than” with “more than”?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph’s adjacency matrix, how do we proceed to prove Savitch’s theorem?

Exercise 5

Prove that, if Rado’s Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 37

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "$\textvisiblespace$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 38

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\sqcup, 1\}$, where "$\sqcup$" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "$\sqcup$" at least once).

1.2) …the machine always writes the "$\sqcup$" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 39

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 40

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 41

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 42

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine always writes the "␣" symbol on the tape.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 43

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 44

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine can only move left.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 45

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 46

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine always writes the "␣" symbol on the tape.

1.2) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 47

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "$\textvisiblespace$" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 48

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine can only move left.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 49

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 50

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) ...the machine always writes the "␣" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 51

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 52

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that...

1.1) ...the machine can only move left.

1.2) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 53

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{␣, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

1.2) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 54

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner, 1\}$, where "$\llcorner$" is the default symbol, such that...

1.1) ...the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "$\llcorner$" at least once).

1.2) ...the machine always writes the "$\llcorner$" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 55

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "$\textvisiblespace$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 56

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine can only move left.

1.2) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 57

Exercise 1

Prove or disprove the computability of function $\overline{\mathtt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language INDEPENDENT SET[1000], defined as follows:

undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of INDEPENDENT SET[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that INDEPENDENT SET[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Radó's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 58

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\text{␣}, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine always writes the "␣" symbol on the tape.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language CLIQUE[1000], defined as follows:

undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of CLIQUE[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that CLIQUE[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "more than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "more than" with "less than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 59

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\llcorner\!\lrcorner, 1\}$, where "$\llcorner\!\lrcorner$" is the default symbol, such that…

1.1) …the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.

1.2) …the machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once).

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **RP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language VERTEX COVER[1000], defined as follows:

undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes.

3.1) Describe a reasonable encoding for an instance of VERTEX COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that VERTEX COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Question sheet 60

Exercise 1

Prove or disprove the computability of function $\overline{\texttt{HALT}}(\mathcal{M}, x)$ that correctly decides the Halting Problem whenever $\mathcal{M}$ is guaranteed to belong to the subclass of Turing machines with one tape and alphabet $\{\textvisiblespace, 1\}$, where "␣" is the default symbol, such that…

1.1) …the machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).

1.2) …the machine can only move left.

Hint — *The Halting Problem is decidable in one of the two subclasses, but not in the other. You can give the uncomputability of the unrestricted Halting Problem for granted.*

Exercise 2

2.1) Define the class **coRP**.

2.2) Show that the definition does not depend on the threshold value, as long as it is strictly between zero and one.

Exercise 3

Consider the language SET COVER[1000], defined as follows:

collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

3.1) Describe a reasonable encoding for an instance of SET COVER[1000] and discuss its size. Remember that "1000" is a constant and does not require to be encoded.

3.2) Prove that SET COVER[1000] $\in$ **P** (although, possibly, with a fairly high exponent).

3.3) Discuss what happens if we replace the words "less than" with "exactly" in the definition: does the problem become harder, easier, trivial (i.e., it has an obvious answer), or does it have the same difficulty?

3.4) What if we replace the words "less than" with "more than"?

Exercise 4

Given that ST-CONNECTIVITY $\in$ DSPACE$\big((\log n)^2\big)$, where n is basically the size of the graph's adjacency matrix, how do we proceed to prove Savitch's theorem?

Exercise 5

Prove that, if Rado's Busy Beaver function $S(n)$ were computable, then the Halting Problem would be decidable.

Hint — *Remember that $S(n)$ is defined as the maximum number of steps that a 1-tape, 2-symbol, n-state TM starting with an all-zero tape can perform before halting.*

Answer guidelines to exercise 1

The exercise proposed two different properties of TMs, and asked whether the Halting Problem is decidable *provided that we knew* that the machine has that property. As explained by the hint, one of the properties made the halting problem computable, the other didn't.

Properties for which the HP remains uncomputable

- The machine never moves right for 5 consecutive times (i.e., after moving right for 4 consecutive steps, it will move left at least once.
  Given a 1-tape, 2-symbol TM $\mathcal{M}$, it is always possible to transform it in a TM $\mathcal{M}'$ that satisfies the above restriction by just adding states. For instance, $\mathcal{M}'$ can perform exactly the same steps of $\mathcal{M}$, each followed by two dummy moves (moving left then right). In this way, $\mathcal{M}'$ performs exactly the same computation of $\mathcal{M}$ (in particular, $\mathcal{M}'$ halts if and only if $\mathcal{M}$ halts) while never moving right for more than two consecutive times, thereby obeying the required restriction.

- The machine never writes "1" for 5 consecutive times (i.e., after writing "1" for 4 consecutive steps, it will write "␣" at least once).
  Same as before, but the dummy states may write a blank and immediately overwrite it with the correct symbol, so that two ones are never written consecutively.

Therefore, if the Halting Problem were decidable for TMs with the requested property, it would also be decidable in the general case by transforming any TM $\mathcal{M}$ into a TM $\mathcal{M}'$ having the requested property.

Properties that make the HP computable

- The tape only has 1000000 cells and is circular, i.e., trespassing the rightmost cell leads to the leftmost one and viceversa;

- the tape only has 1000000 cells and the machine halts when it tries to trespass the rightmost or leftmost cell.
  In the two cases above, the machine has a finite tape, therefore a finite number of configurations (symbols on tape, current position, current state). Either the machine halts, or it is going to repeat at least one configuration. Just simulate it until then.

- The machine always writes the "␣" symbol on the tape.
  The machine overwrites the input with blanks. Two things can happen:

  - either the input is eventually completely overwritten, in which case the machine's behavior becomes only determined by the state (since the input will always be blank); at that point, if the machine doesn't halt before repeating a state, it will run forever;
  - or the machine never overwrites the whole input string with blanks; in such case, either it re-enters the same configuration twice, or it wanders far away from the untouched input (in which case it won't halt because of the limited number of states).

- The machine can only move left.
  If the machine has n states, either it halts within n steps after leaving the written part of the tape, or it will repeat a state and keep reading blanks, therefore going on forever.

Observations

A formal proof wasn't required: observations such as the ones written above were sufficient to get full marks.

Note that the exercise didn't ask about the decidability of the mentioned property, but to *assume* that the machine to be tested had it.

Answer guidelines to exercise 2

2.1 — Any definition of **RP** or **coRP** (non-deterministic, probabilistic) is fine.
2.2 — The answer should mention, in a way or another, the repetition of the algorithm for a number of times N sufficient to increase the probability of true positives (or true negatives, in the **coRP** case), above any threshold ε, provided that $0 < \varepsilon < 1$.

Answer guidelines to exercise 3

A particular case of a well known NP-complete problem was given:

- INDEPENDENT SET[1000] — undirected graphs with at least one independent set (a completely disconnected subgraph) of more than 1000 nodes;

- CLIQUE[1000] — undirected graphs with at least one clique (a completely connected subgraph) of more than 1000 nodes;

- VERTEX COVER[1000] — undirected graphs with at least one vertex cover (a subset of nodes such as every edge of the graph has an endpoint in it) of less than 1000 nodes;

- SET COVER[1000] — collections of subsets of $\{1, \dots, n\}$ with at least one cover (a subcollection whose union is the whole $\{1, \dots, n\}$) of less than 1000 sets.

Note that the restriction is about the size of the requested subset (of graph nodes or of subsets).
3.1 — In the first three cases, a reasonable encoding of a graph is its adjacency matrix, or its adjacency list, possibly complemented with the size of the graph; in the last case, the list of elements of each set.
3.2 — In the first two cases (INDEPENDENT SET and CLIQUE), finding a solution with 1001 nodes is enough to answer the question. Therefore, we need to try all possible subsets of nodes of size 1001. In n is the number of nodes in the graph, then we must iterate through $\binom{n}{1000} = O(n^{1000})$ different subsets (a number which is polynomial in the size of the graph's encoding described at point 2.1), and for each of them check whether they form an independent set/clique, which can be again done in polynomial time.
In the latter two cases, since they require a subset of size less than 1000, we just need to check for all subsets of size 999.

3.3 — The difficulty doesn't change very much. In all cases we have to iterate among all subsets of size 1000, making the first two problems slightly easier, the two latter problems slightly harder — still polynomial, though.
3.4 — The problems become trivial: every graph has an independent set of size *less than* 1000 (take just one random node, or two nodes that aren't connected if you don't like a one-node solution) or a clique of size *less than* 1000 (take just one random node, or two nodes that are connected if you don't like a one-node solution). A graph has a vertex cover of *more than* 1000 nodes if and only if it has more than 1000 nodes itself (the cover made of all nodes). A collection of sets has a cover of $\{1, \dots, n\}$ of size *more than* 1000 sets if and only if it has more than 1000 nodes itself (and its union is the whole $\{1, \dots, n\}$).
In other words finding an arbitrarily small independent set or clique, or an arbitrarily large vertex cover or set cover, is hardly a problem.

Answer guidelines to exercise 4

See a proof of Savitch's Theorem. Not all details were necessary to get full marks, only a reasonably comprehensive outline of the proof.

Observe that the exercise did *not* ask to prove that STCON $\in$ DSPACE$\big((\log n)^2\big)$, but to assume it was already proved and proceed from that point on.

Answer guidelines to exercise 5

See Theorem 7 in the lecture notes.