Computability and Computational Complexity, A.Y. 2018–2019

Second partial test — Computational complexity

Mauro Brunato

Friday, december 21, 2018

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- Answer guidelines

Question sheet 1

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

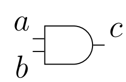
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 2

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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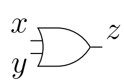

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> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 3

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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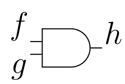

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3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 4

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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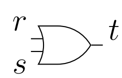

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Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 5

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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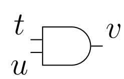

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3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

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Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 6

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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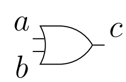

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3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 7

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

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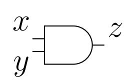

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Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 8

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

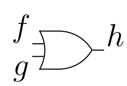
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



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3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 9

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

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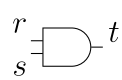

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3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

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Question sheet 10

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

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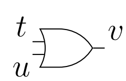
Consider an OR gate with the following inputs and outputs:



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Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 11

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

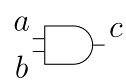
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



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Exercise 4

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Question sheet 12

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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Exercise 2

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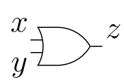

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3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 13

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

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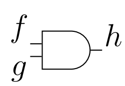
Consider an AND gate with the following inputs and outputs:



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Question sheet 14

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1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

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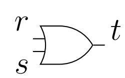

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Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 15

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

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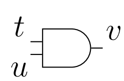

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3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 16

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

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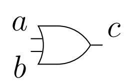

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3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 17

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

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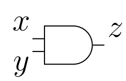
Consider an AND gate with the following inputs and outputs:



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3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 18

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

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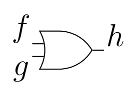
Consider an OR gate with the following inputs and outputs:



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3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 19

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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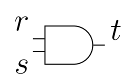

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Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 20

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

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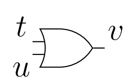

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Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 21

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

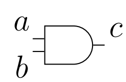
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 22

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

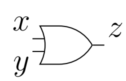
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 23

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

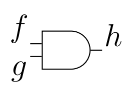
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 24

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

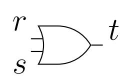
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 25

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

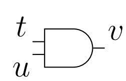
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 26

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

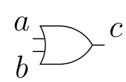
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 27

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

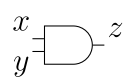
1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 28

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

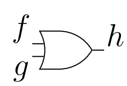
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 29

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

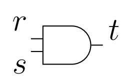
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 30

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

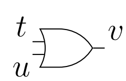
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 31

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

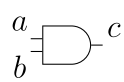
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{PSPACE} \subseteq \mathbf{EXP}$.

Question sheet 32

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

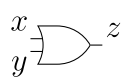
1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 33

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

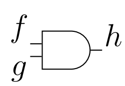
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 34

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

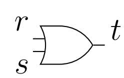
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 35

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

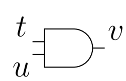
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 36

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

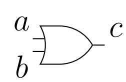
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 37

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

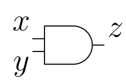
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 38

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

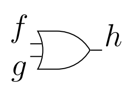
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 39

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

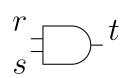
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{PSPACE} \subseteq \mathbf{EXP}$.

Question sheet 40

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

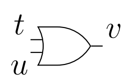
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

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3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 41

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

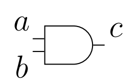
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 42

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

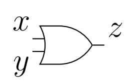
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 43

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

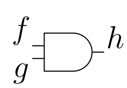
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 44

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

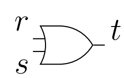
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 45

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

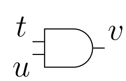
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 46

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

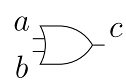
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 47

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

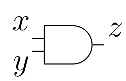
1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

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> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 48

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

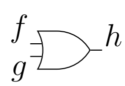
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 49

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

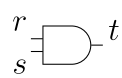
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means “a and b know each other”, and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 50

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

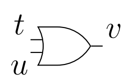
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 51

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

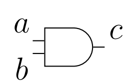
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means “a knows b”, and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 52

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

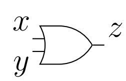
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 53

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

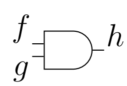
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 54

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

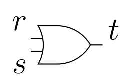
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 55

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

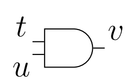
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

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3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 56

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

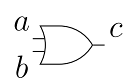
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 57

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

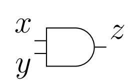
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



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3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 58

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

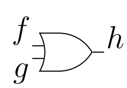
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

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> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 59

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

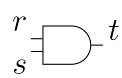
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



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> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 60

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

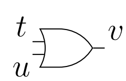
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means “a knows b”, and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 61

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

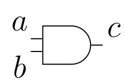
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 62

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

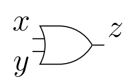
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 63

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

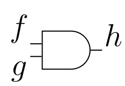
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means “a knows b”, and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 64

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

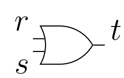
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 65

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

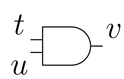
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 66

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

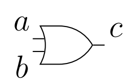
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means “a knows b”, and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 67

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

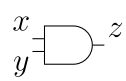
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

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> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 68

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

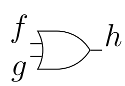
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 69

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

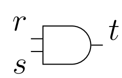
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 70

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

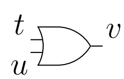
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 71

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

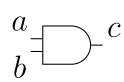
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 72

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

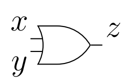
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 73

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

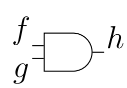
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 74

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

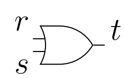
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

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> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 75

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

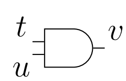
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means “a knows b”, and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 76

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

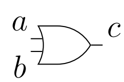
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



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Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 77

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

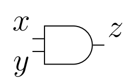
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



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Exercise 3

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> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 78

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

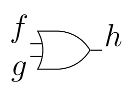
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 79

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

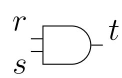
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



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Exercise 3

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> Given a list of unordered pairs of persons, where $\{a, b\}$ means “a and b know each other”, and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 80

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

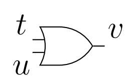
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 81

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

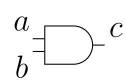
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a,b,c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a,b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 82

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

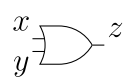
1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 83

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

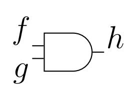
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{PSPACE} \subseteq \mathbf{EXP}$.

Question sheet 84

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

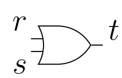
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 85

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

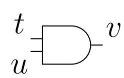
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 86

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

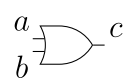
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 87

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

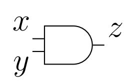
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 88

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

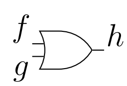
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 89

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

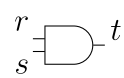
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 90

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

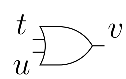
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 91

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

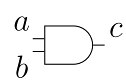
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 92

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

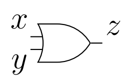
1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 93

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

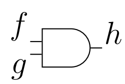
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 94

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

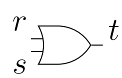
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 95

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

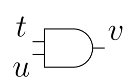
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 96

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

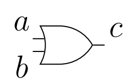
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 97

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

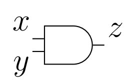
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



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> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 98

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

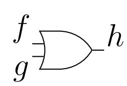
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

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> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 99

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

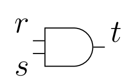
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

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> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 100

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

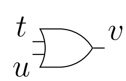
1.2) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 101

Exercise 1

Consider the following languages:

1.1) SAT (satisfiable CNF formulae);

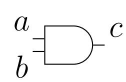
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{L} \subseteq \mathbf{P}$.

Question sheet 102

Exercise 1

Consider the following languages:

1.1) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause);

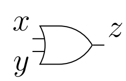
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Question sheet 103

Exercise 1

Consider the following languages:

1.1) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected);

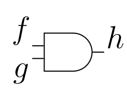
1.2) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means “a and b know each other”, and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 104

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

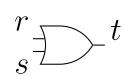
1.2) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means "a and b can be used in the same recipe", and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 105

Exercise 1

Consider the following languages:

1.1) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path);

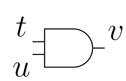
1.2) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 106

Exercise 1

Consider the following languages:

1.1) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t);

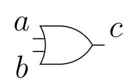
1.2) SAT (satisfiable CNF formulae).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(a, b, c)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NL** $\subseteq$ **NP**.

Question sheet 107

Exercise 1

Consider the following languages:

1.1) 2-VERTEX COLORING (undirected graph whose vertices can partitioned in two subsets each of which is an independent set);

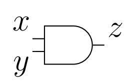
1.2) 3-SAT (satisfiable CNF formulae with at most 3 literals per clause).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(x, y, z)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **PSPACE** $\subseteq$ **EXP**.

Question sheet 108

Exercise 1

Consider the following languages:

1.1) 2-SAT (satisfiable CNF formulae with at most 2 literals per clause);

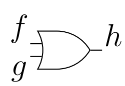
1.2) INDEPENDENT SET (undirected graph G and number k such that at least k vertices are mutually disconnected).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(f, g, h)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of ordered pairs of persons, where (a, b) means "a knows b", and a number k, is there an individual who is known by at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **NPSPACE** $\subseteq$ **NEXP**.

Question sheet 109

Exercise 1

Consider the following languages:

1.1) CLIQUE (undirected graph G and number k such that at least k vertices are mutually connected);

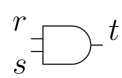
1.2) GRAPH CONNECTIVITY (undirected graphs where every pair of nodes is connected by a path).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be "Yes", "Likely", "Unlikely", or "No", followed by a short motivation.

Exercise 2

Consider an AND gate with the following inputs and outputs:



Show how to determine a CNF formula $f(r, s, t)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of persons, where $\{a, b\}$ means "a and b know each other", and a number k, is there an individual who knows at least k other people?

3.2) Prove that if the problem above is **NP**-complete then **P** $=$ **NP**.

Exercise 4

Prove that **L** $\subseteq$ **P**.

Question sheet 110

Exercise 1

Consider the following languages:

1.1) 3-VERTEX COLORING (undirected graph whose vertices can partitioned in three subsets each of which is an independent set);

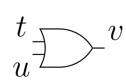
1.2) ST-CONNECTIVITY (directed graph and two nodes s, t such that there is a path from s to t).

Discuss their inclusion in **NP**, **RP**, **coNP**, **coRP**.

For each language and class, the discussion can be “Yes”, “Likely”, “Unlikely”, or “No”, followed by a short motivation.

Exercise 2

Consider an OR gate with the following inputs and outputs:



Show how to determine a CNF formula $f(t, u, v)$ that is satisfiable by all feasible combinations of input and output truth values, and only by them.

Exercise 3

3.1) Prove that the following problem is **NP**:

> Given a list of unordered pairs of cooking ingredients, where $\{a, b\}$ means “a and b can be used in the same recipe”, and a number k, is there an ingredient that is compatible with at least k others?

3.2) Prove that if the problem above is **NP**-complete then $\mathbf{P} = \mathbf{NP}$.

Exercise 4

Prove that $\mathbf{NL} \subseteq \mathbf{NP}$.

Answer guidelines to exercise 1

Every question sheet had two languages:

- an **NP**-complete language L chosen among SAT, 3-SAT, INDEPENDENT SET and 3-VERTEX COLORING.
  - $L \in$ **NP** could be proven by using a solution as certificate, but even a simple "it is well known" was accepted as an answer.
  - $L \in$ **RP** is very unlikely. In fact, L is known to be **NP**-complete, hence every language $L' \in$ **NP** could be polynomially reduced to $L \in$ **RP**, meaning that **NP** $=$ **RP**, which is considered to be very improbable.
  - The same reasoning can be used to suggest that both $L \in$ **coRP** and $L \in$ **coNP** are unlikely as well.
- a **P** language L chosen among ST-CONNECT, 2-VERTEX COLORING, 2-SAT and GRAPH CONNECTIVITY.
  - Since $L \in$ **P**, and **P** is included in all classes mentioned in the exercise, the answer is uniformly "yes".

Other possible answers

Recalling the definitions is OK. For instance:

- one could say that CLIQUE is unlikely to be in **RP** because there is no nonzero lower bound ε on the frequency of correct solutions among all possible subsets, *and that no substantially better certificates are known* (this part is important),
- or that SAT is unlikely to be in **coNP** because there is no known polynomial certification of unsatisfiability.

(remember, these are only a few examples)

Common errors

- "We know that $L \in$ **NP**, but **RP** $\subseteq$ **NP**, therefore L cannot possibly be in **RP**." (wat)
- "**RP** $\supseteq$ **NP**." (it's the other way round)

- "Since $L \in$ **NP**, then $x \in L$ has at least one valid certificate, therefore the ratio of valid certificates over the total number is not zero, and then $L \in$ **RP**." (then why bother defining **RP** at all? The ratio might be vanishingly small, hence not bounded, for increasing input sizes)

- "Since $L \in$ **NP**, and by setting $\varepsilon = 0$ we get **RP** $=$ **NP**, then $L \in$ **RP**." (then why bother defining **RP** at all? Anyway, the definition requires $\varepsilon > 0$)

- "Since $L \in$ **NP**, then it cannot be in **coNP** because they are complementary." (they aren't: their intersection contains at least **P**, and possibly many more things)

Answer guidelines to exercise 2

See Fig. 2.2 of the lecture notes.

Answer guidelines to exercise 3

1. All proposed languages are polynomial, solved by a linear scan of the inputs and keeping counters. They are all reducible to finding a node in a (un)directed graph with a (—/in/out)degree larger than k. Since the language is in **P**, it is a fortiori in **NP**.

2. If the language were **NP**-complete, then every language in **NP** would be polynomially reducible to a polynomial language, hence **P** $=$ **NP**.

Common errors

Rather than understanding the problem described in the text, somebody might automatically recognize a graph problem and say "Then it must be a sort of CLIQUE or INDSET, thus **NP**-complete" or, if lucky, "This is just STCON or CONNECTIVITY" (still wrong, but at least they are **P**). Understanding the problem actually reveals that the solution is far simpler than those.

Answer guidelines to exercise 4

One of the following four inclusions had to be proved:

- **L** $\subseteq$ **P**

- **NL** $\subseteq$ **NP**
- **PSPACE** $\subseteq$ **EXP**
- **NPSPACE** $\subseteq$ **NEXP**

All proofs follow a similar scheme: the time constraint cannot be above an exponential function of the space constraint, otherwise a machine configuration would be repeated and the machine wouldn't halt. See, for instance, Theorem 23 in the lecture notes.

Common errors

Remember that **L** is logarithmic *space* (time would not make sense, given that we expect at least linear time in order to be able to scan the input).