Computability and Computational Complexity, A.Y. 2018–2019

First partial test — Computability

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Contents

Question sheet 1

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice’s theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with more states than alphabet symbols;

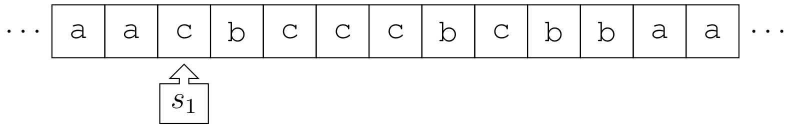
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

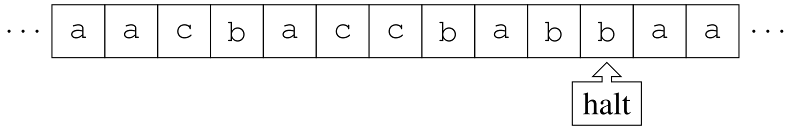
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where ‘`a`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless ‘`a`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every ‘`c`’ that immediately follows ‘`b`’ must be replaced with ‘`a`’ (i.e., every sequence ‘`bc`’ must become ’`ba`’).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`a`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`bcbbccbbbccc`”

Question sheet 2

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

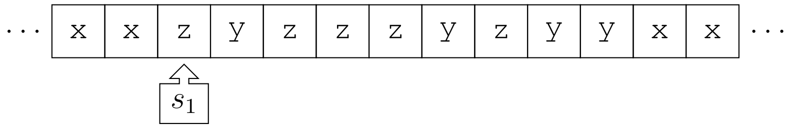
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

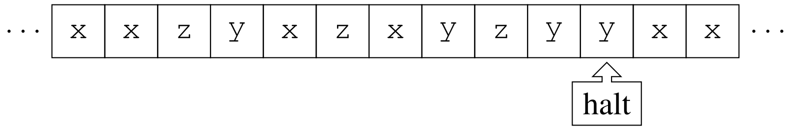
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyzyyzz`"

Question sheet 3

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with less states than alphabet symbols;

1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case

···	1	1	3	2	3	3	3	2	3	2	2	1	1	···
			s_1											

the final configuration should be

···	1	1	3	1	3	3	3	2	3	1	2	1	1	···
											halt			

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333223323`"

Question sheet 4

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

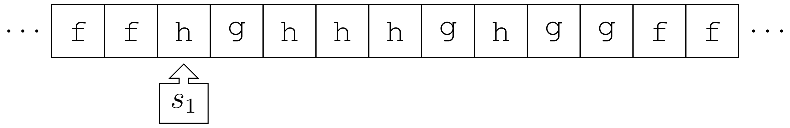
1.3) M recognizes Turing machines with less than 100 states.

Exercise 2

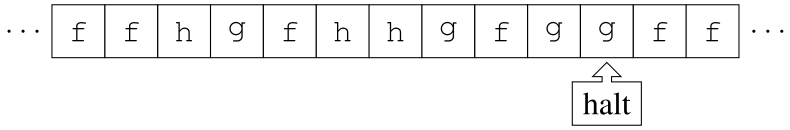
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 5

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

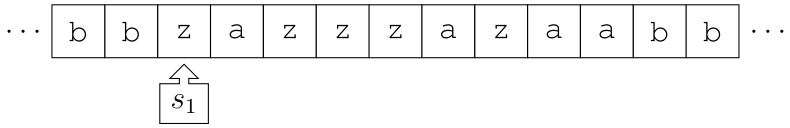
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

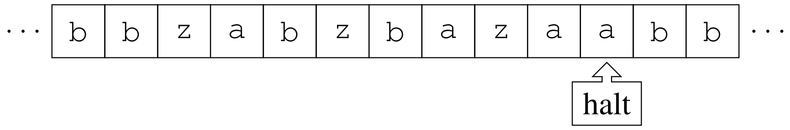
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 6

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

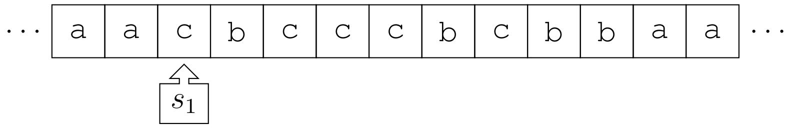
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

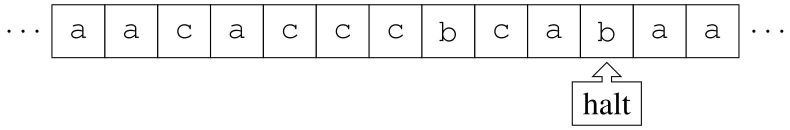
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbcbbcc`"

Question sheet 7

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

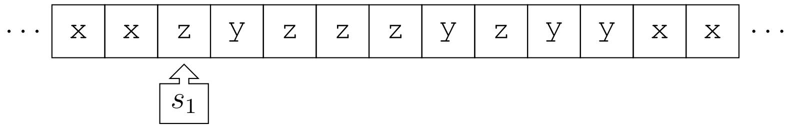
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

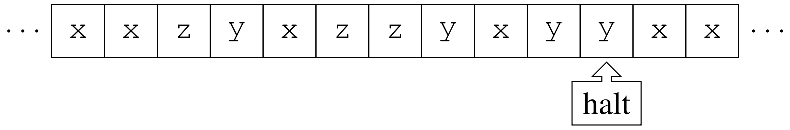
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{x}, \texttt{y}, \texttt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{y}, \texttt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyyzzyz`"

Question sheet 8

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on input `11111`;

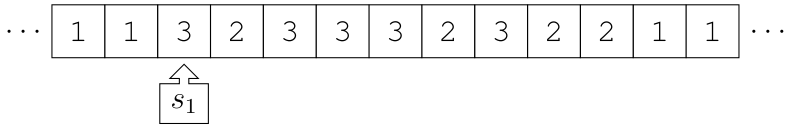
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

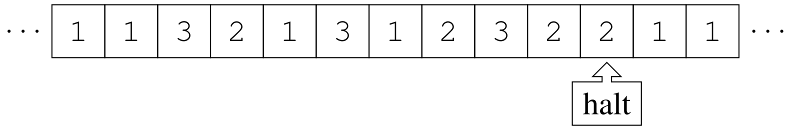
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	2	1	3	1	2	3	2	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`223322233323`"

Question sheet 9

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

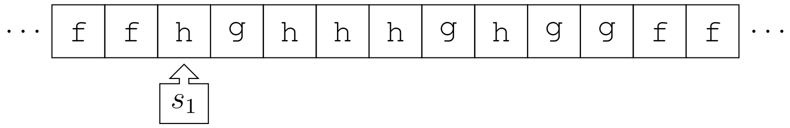
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

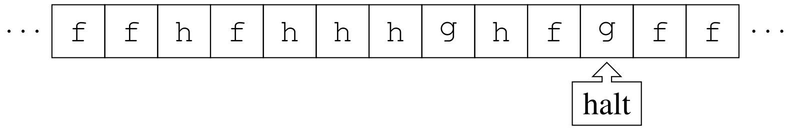
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where ‘`f`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless ‘`f`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol ‘`g`’ must be replaced with ‘`f`’ (i.e., the first ‘`g`’ must become ‘`f`’, the second must not be changed, the third must become ‘`f`’…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`f`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`ghgghhggghhh`”

Question sheet 10

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

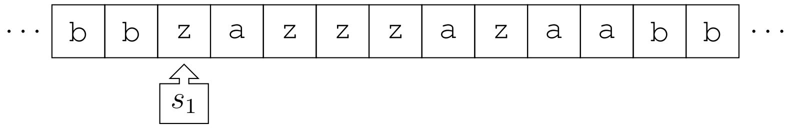
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

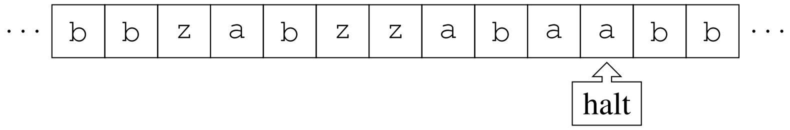
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where ‘`b`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless ‘`b`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every ‘`z`’ that immediately follows ‘`a`’ must be replaced with ‘`b`’ (i.e., every sequence ‘`az`’ must become ’`ab`’).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`b`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`aaazzzazaazz`”

Question sheet 11

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M halts after more than 100 steps when executed on an empty tape;

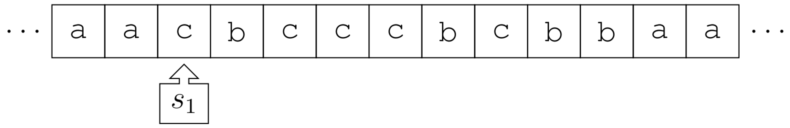
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

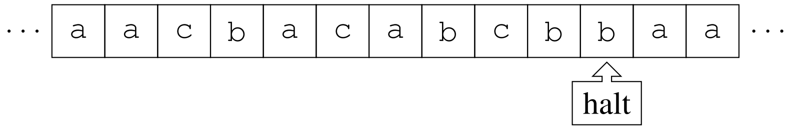
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 12

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M visits every state at most ten times when executed on input `00000`;

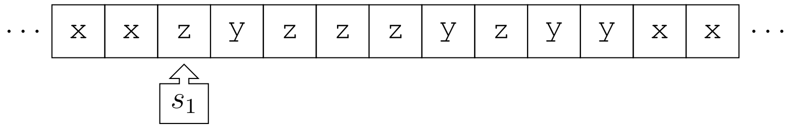
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

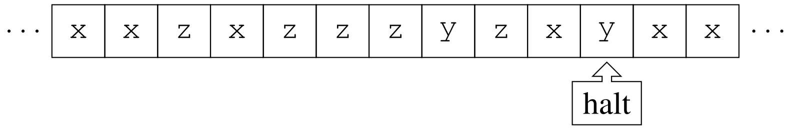
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyzzyyyzzzyz`"

Question sheet 13

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M recognizes Turing machines with less states than alphabet symbols;

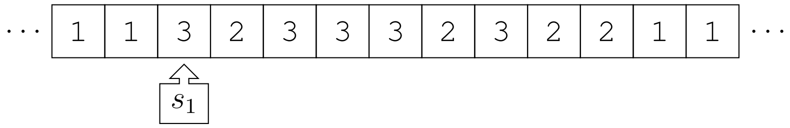
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

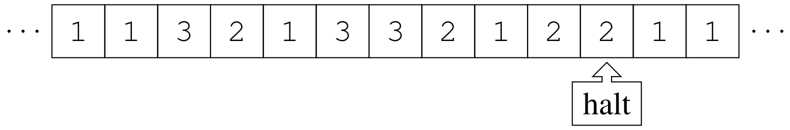
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 14

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

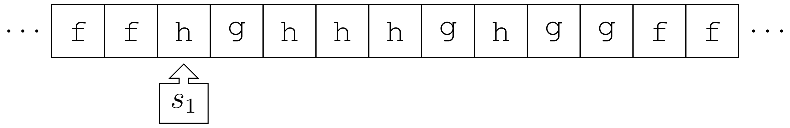
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

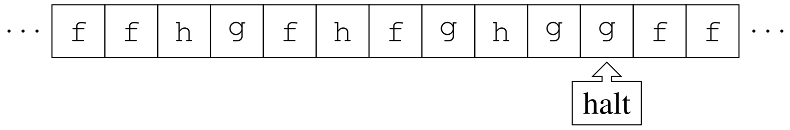
- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	f	f	h	g	h	h	h	g	h	g	g	f	f	···

the final configuration should be



···	f	f	h	g	f	h	f	g	h	g	g	f	f	···

You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhghgghh`"

Question sheet 15

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M visits every state at most ten times when executed on input `00000`;

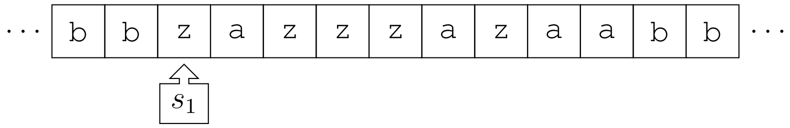
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

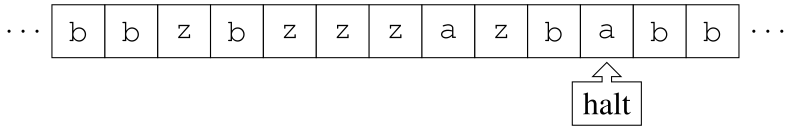
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 16

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

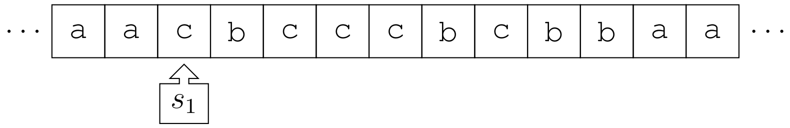
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

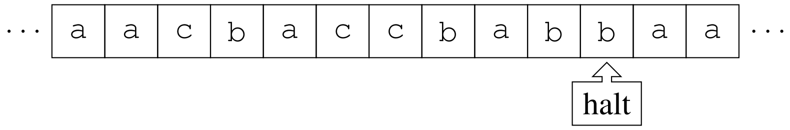
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`bbccbbbcccbc`”

Question sheet 17

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

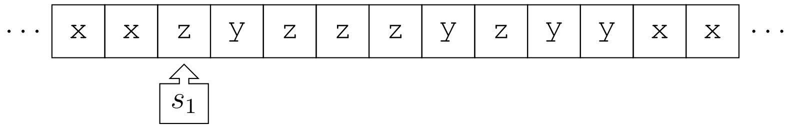
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

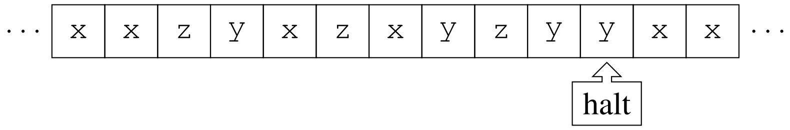
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yzyyzzyyyzzz`"

Question sheet 18

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with less states than alphabet symbols;

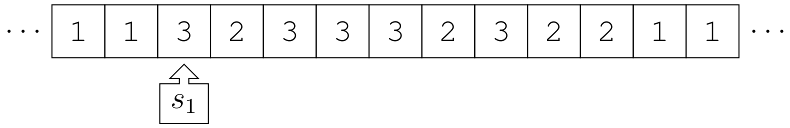
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

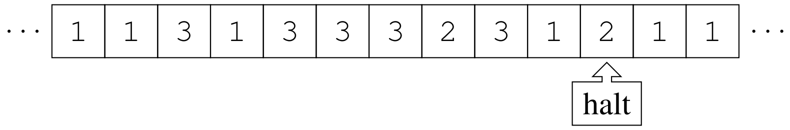
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333232233`"

Question sheet 19

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

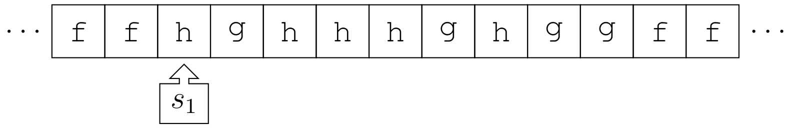
1.3) M recognizes Turing machines with less than 100 states.

Exercise 2

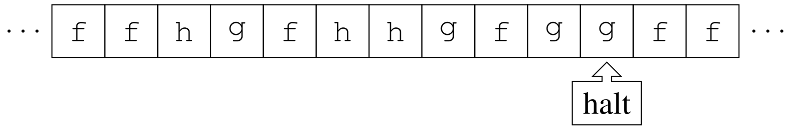
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhgghhgh`"

Question sheet 20

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

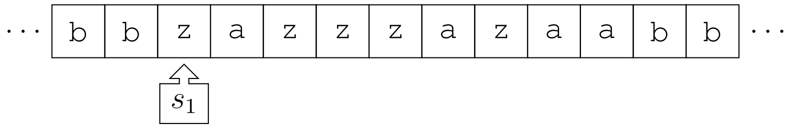
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

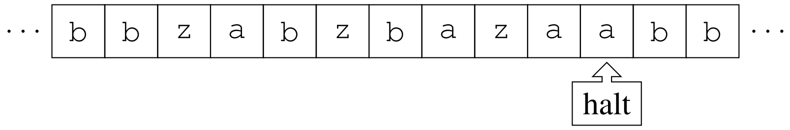
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Question sheet 21

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

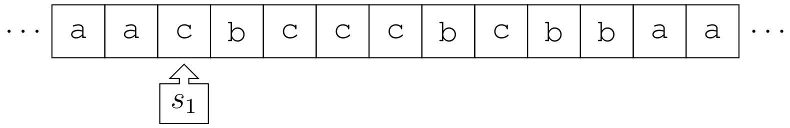
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

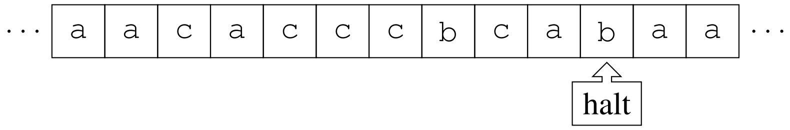
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{b}, \texttt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bcbbccbbbccc`"

Question sheet 22

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

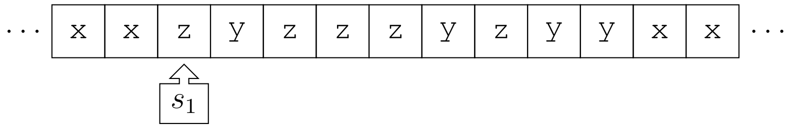
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

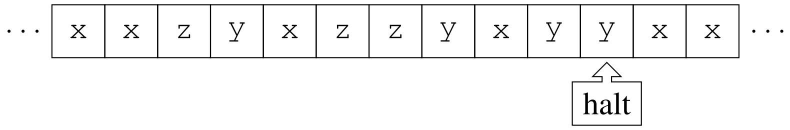
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where ‘`x`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless ‘`x`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every ‘`z`’ that immediately follows ‘`y`’ must be replaced with ‘`x`’ (i.e., every sequence ‘`yz`’ must become ’`yx`’).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`x`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yyyzzzyzyyzz`”

Question sheet 23

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M halts after more than 100 steps when executed on an empty tape;

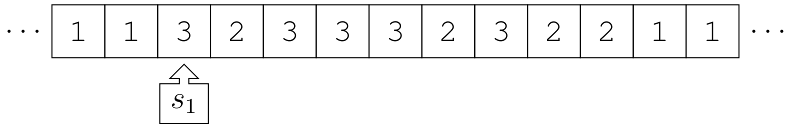
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

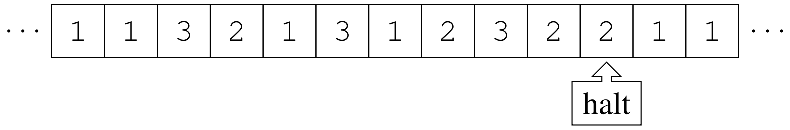
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333223323`"

Question sheet 24

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

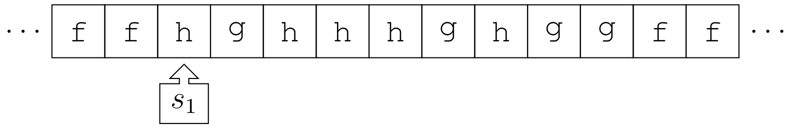
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

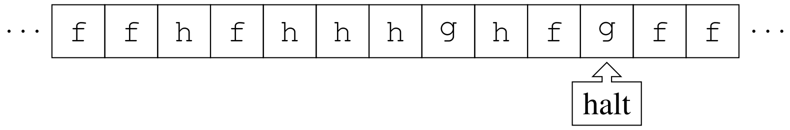
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 25

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

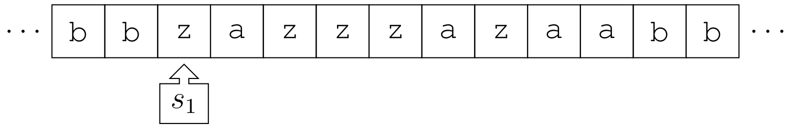
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

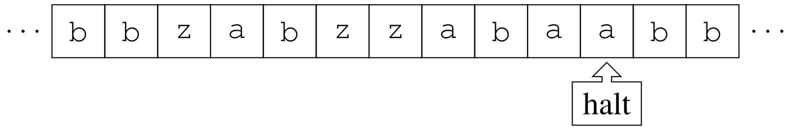
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 26

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

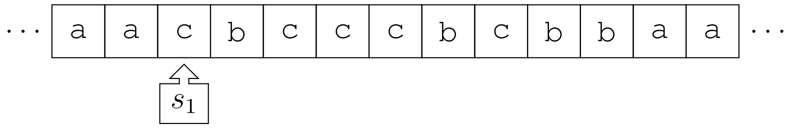
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

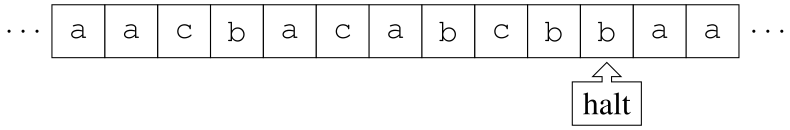
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where ‘`a`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless ‘`a`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol ‘`c`’ must be replaced with ‘`a`’ (i.e., the first ‘`c`’ must not be changed, the second must become ‘`a`’, the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`a`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`bbbcccbcbbcc`”

Question sheet 27

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M visits every state at most ten times when executed on input `00000`;

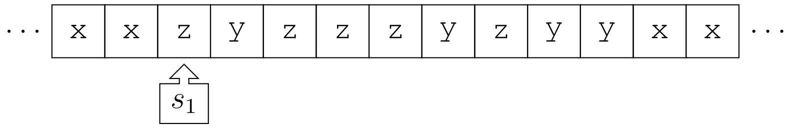
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

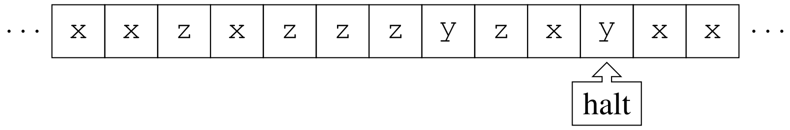
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{x}, \texttt{y}, \texttt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{y}, \texttt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyyzzyz`"

Question sheet 28

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

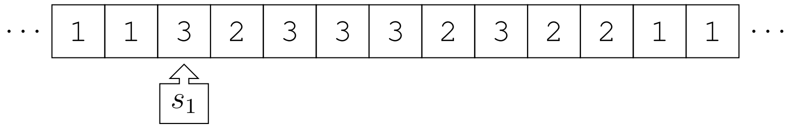
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

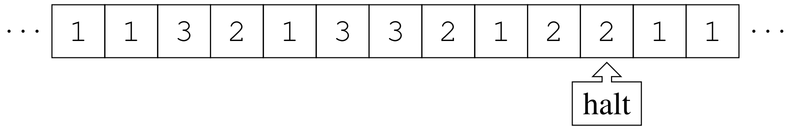
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`223322233323`"

Question sheet 29

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

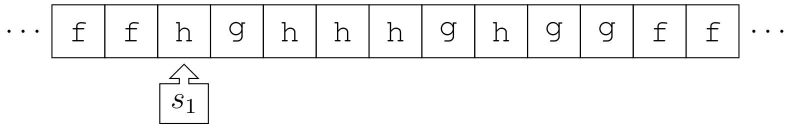
1.3) M recognizes Turing machines with less than 100 states.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

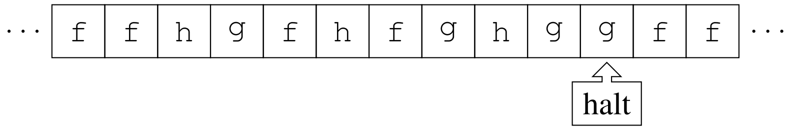
- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	f	f	h	g	h	h	h	g	h	g	g	f	f	···

the final configuration should be



···	f	f	h	g	f	h	f	g	h	g	g	f	f	···

You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ghgghhggghhh`"

Question sheet 30

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with exactly 100 states;

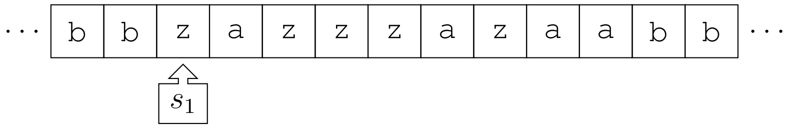
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

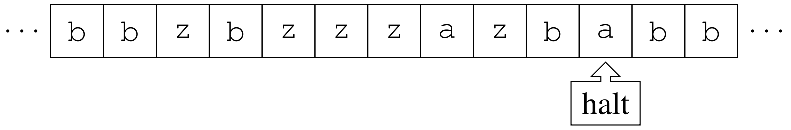
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzazaazz`"

Question sheet 31

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with more states than alphabet symbols;

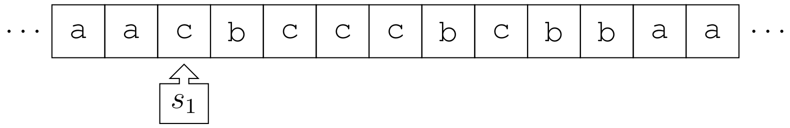
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

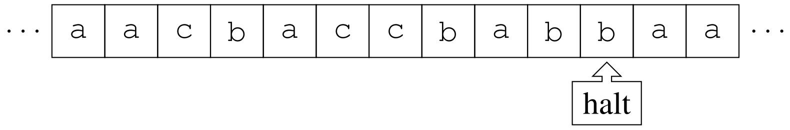
- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	a	a	c	b	c	c	c	b	c	b	b	a	a	···

the final configuration should be



···	a	a	c	b	a	c	c	b	a	b	b	a	a	···

You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 32

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

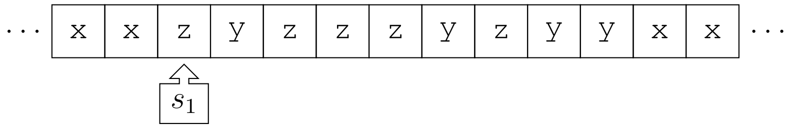
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

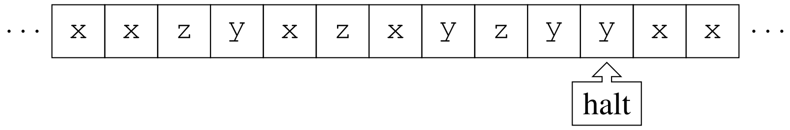
- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	x	x	z	y	z	z	z	y	z	y	y	x	x	⋯

the final configuration should be



⋯	x	x	z	y	x	z	x	y	z	y	y	x	x	⋯

You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyzzyyyzzzyz`"

Question sheet 33

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

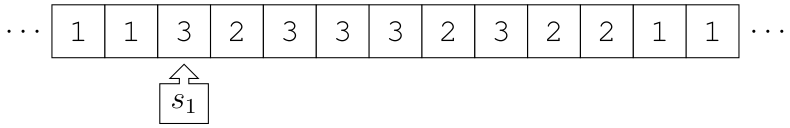
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

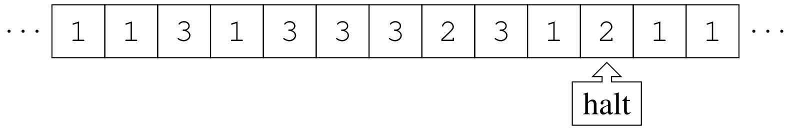
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 34

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice’s theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with less than 100 states;

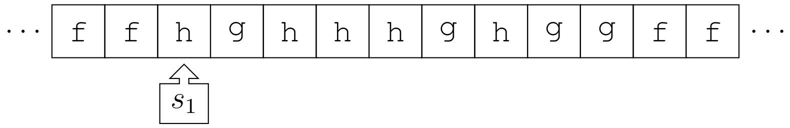
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

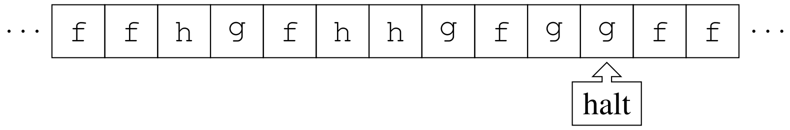
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where ‘`f`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless ‘`f`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every ‘`h`’ that immediately follows ‘`g`’ must be replaced with ‘`f`’ (i.e., every sequence ‘`gh`’ must become ’`gf`’).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`f`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`ggghhhghgghh`”

Question sheet 35

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

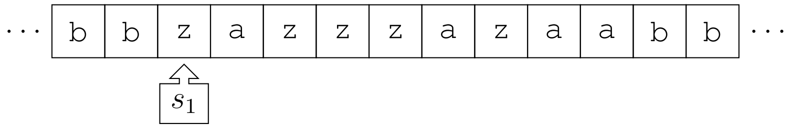
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

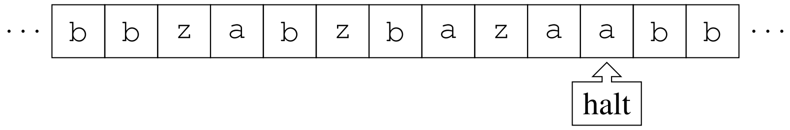
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 36

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with more states than alphabet symbols;

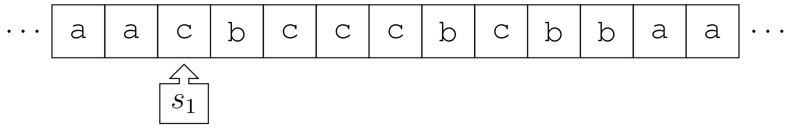
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

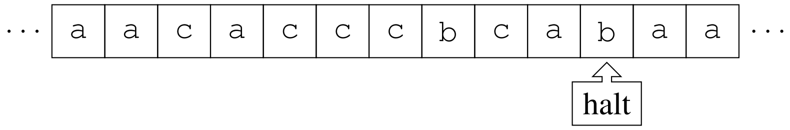
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbccbbbcccbc`"

Question sheet 37

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

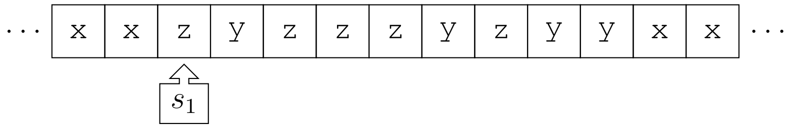
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

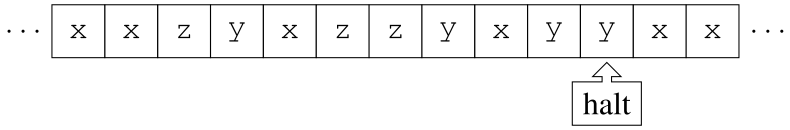
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{x}, \texttt{y}, \texttt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{y}, \texttt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yzyyzzyyyzzz`"

Question sheet 38

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on input `11111`;

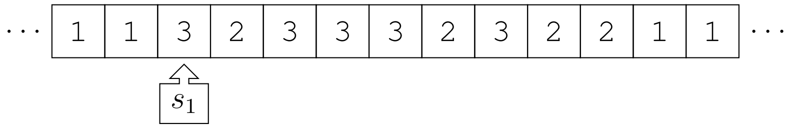
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

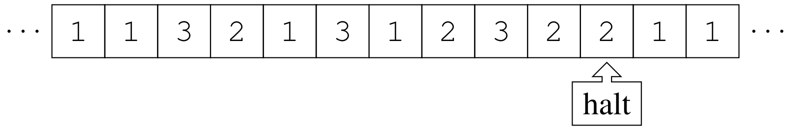
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`222333232233`”

Question sheet 39

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M halts after more than 100 steps when executed on an empty tape;

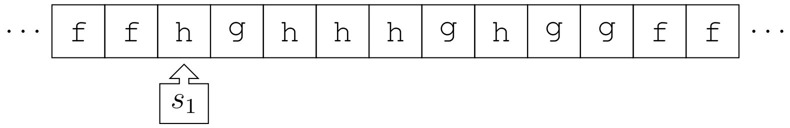
1.3) M recognizes Turing machines with less than 100 states.

Exercise 2

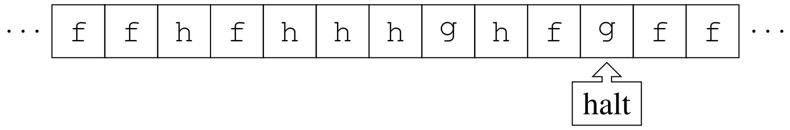
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhgghhgh`"

Question sheet 40

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

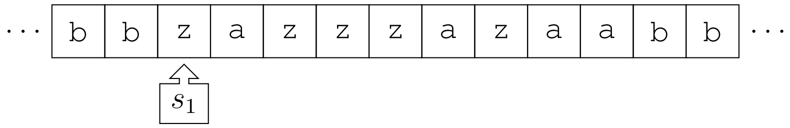
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

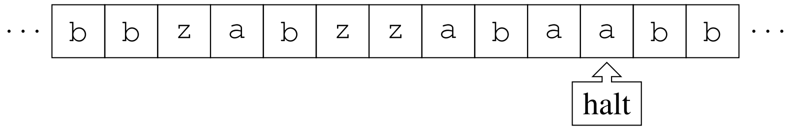
- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	b	b	z	a	z	z	z	a	z	a	a	b	b	···

the final configuration should be



···	b	b	z	a	b	z	z	a	b	a	a	b	b	···

You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Question sheet 41

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M recognizes Turing machines with more states than alphabet symbols;

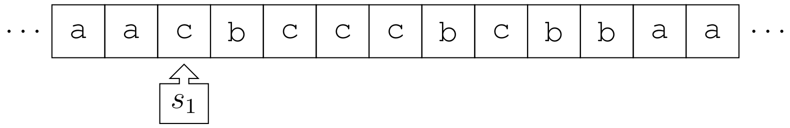
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

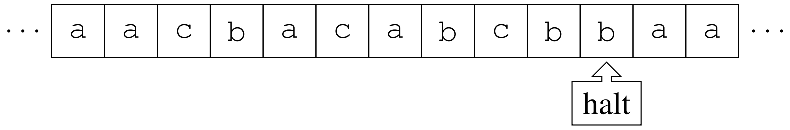
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bcbbccbbbccc`"

Question sheet 42

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

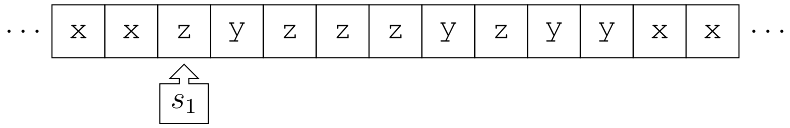
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

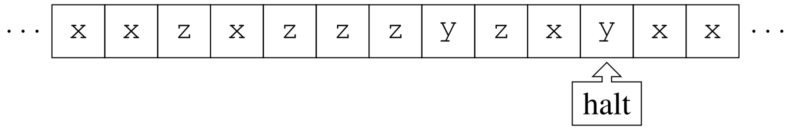
- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	x	x	z	y	z	z	z	y	z	y	y	x	x	···

the final configuration should be



···	x	x	z	x	z	z	z	y	z	x	y	x	x	···

You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyzyyzz`"

Question sheet 43

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on an empty tape;

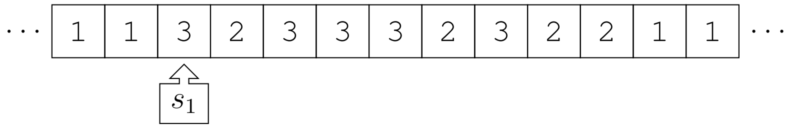
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

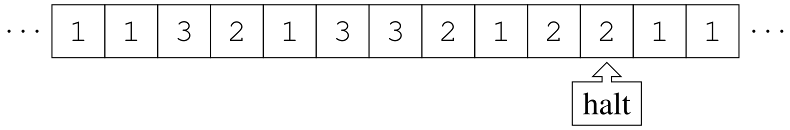
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where `1` is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333223323`"

Question sheet 44

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M recognizes Turing machines with less than 100 states;

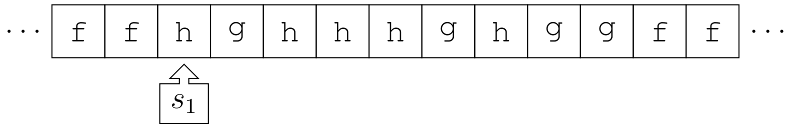
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

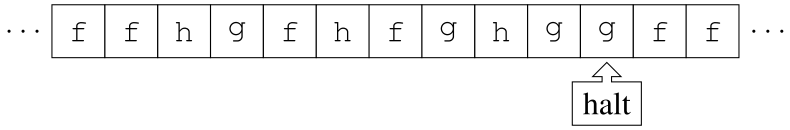
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 45

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with exactly 100 states;

1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case

⋯	b	b	z	a	z	z	z	a	z	a	a	b	b	⋯
			s_1											

the final configuration should be

⋯	b	b	z	b	z	z	z	a	z	b	a	b	b	⋯
											halt			

You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 46

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

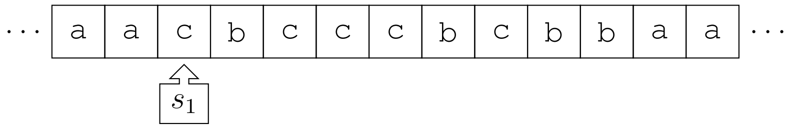
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

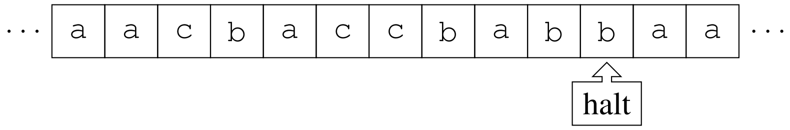
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where ‘a’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless ‘a’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every ‘c’ that immediately follows ‘b’ must be replaced with ‘a’ (i.e., every sequence ‘bc’ must become ’ba’).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘a’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“bbbcccbcbbcc”

Question sheet 47

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M recognizes Turing machines with more than 100 states;

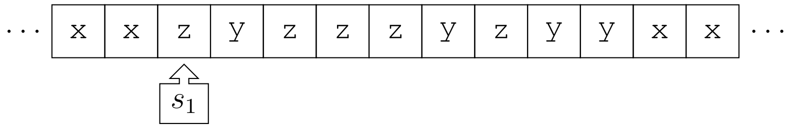
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

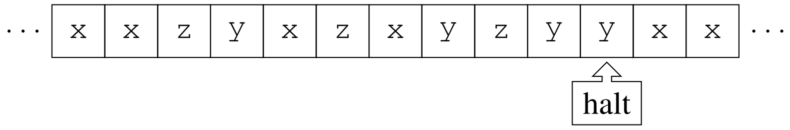
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyyzzyz`"

Question sheet 48

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with less states than alphabet symbols;

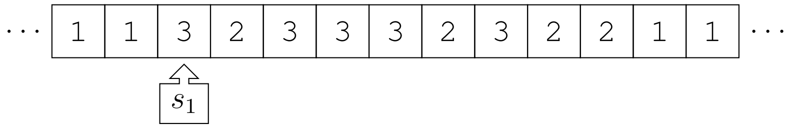
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

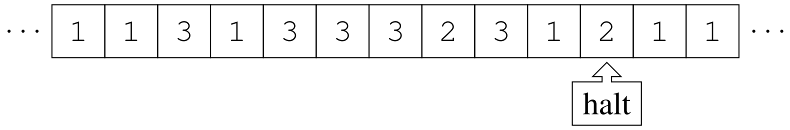
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	1	3	3	3	2	3	1	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`223322233323`"

Question sheet 49

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

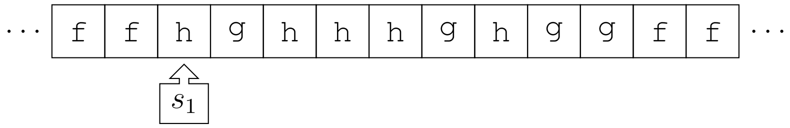
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

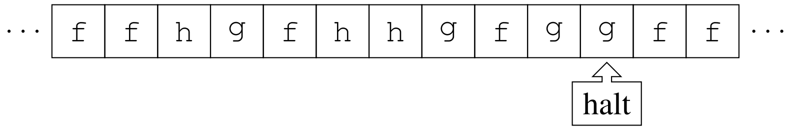
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ghgghhggghhh`"

Question sheet 50

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

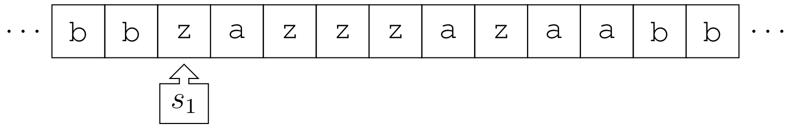
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

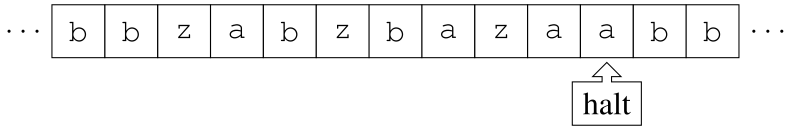
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`aaazzzazaazz`”

Question sheet 51

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M halts after more than 100 steps when executed on an empty tape;

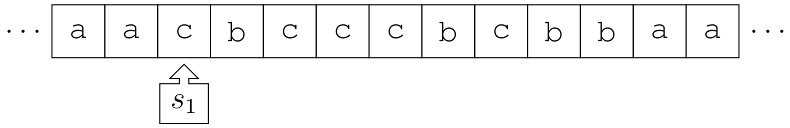
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

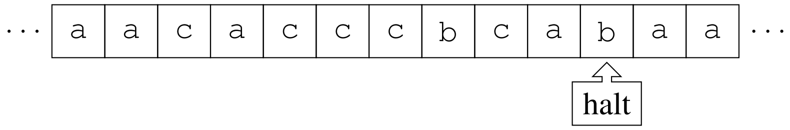
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 52

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

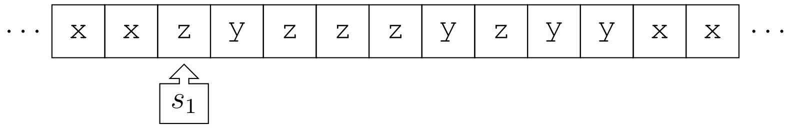
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

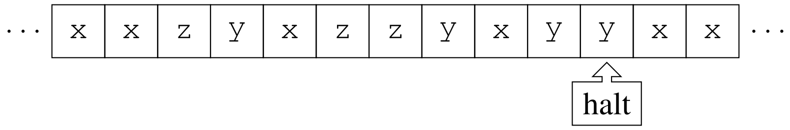
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyzzyyyzzzyz`"

Question sheet 53

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on input `11111`;

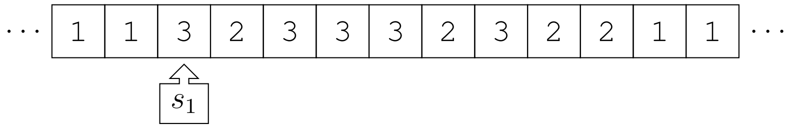
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

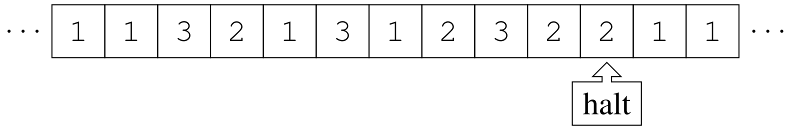
- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	1	1	3	2	3	3	3	2	3	2	2	1	1	⋯

the final configuration should be



⋯	1	1	3	2	1	3	1	2	3	2	2	1	1	⋯

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 54

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M visits every state at most ten times when executed on input `00000`;

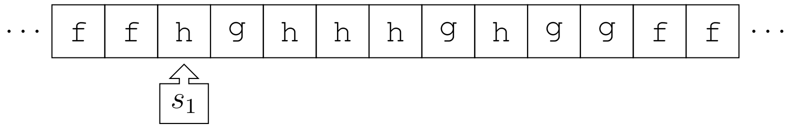
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

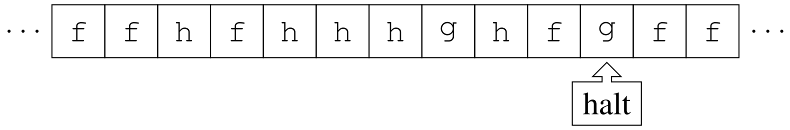
- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	f	f	h	g	h	h	h	g	h	g	g	f	f	···

the final configuration should be



···	f	f	h	f	h	h	h	g	h	f	g	f	f	···

You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhghgghh`"

Question sheet 55

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M halts after more than 100 steps when executed on an empty tape;

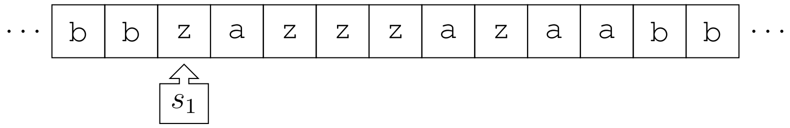
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

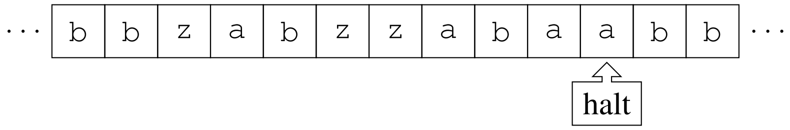
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 56

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M recognizes Turing machines with more states than alphabet symbols;

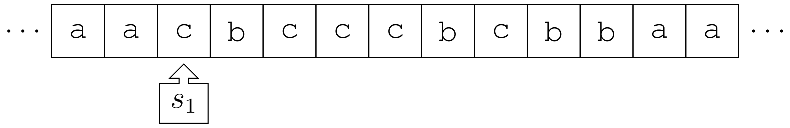
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

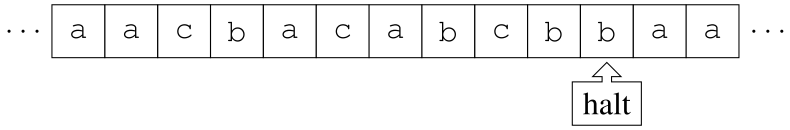
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbccbbbcccbc`"

Question sheet 57

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

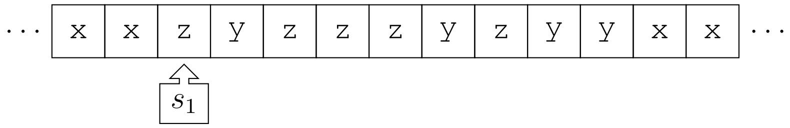
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

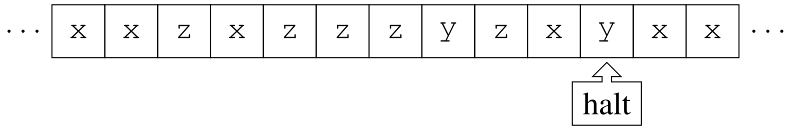
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yzyyzzyyyzzz`"

Question sheet 58

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on an empty tape;

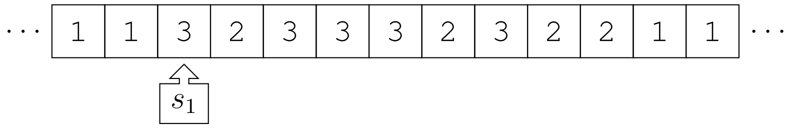
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

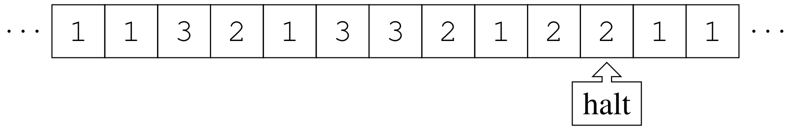
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{1, 2, 3\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{2, 3\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333232233`"

Question sheet 59

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M recognizes Turing machines with less than 100 states;

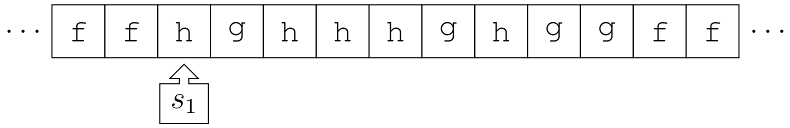
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

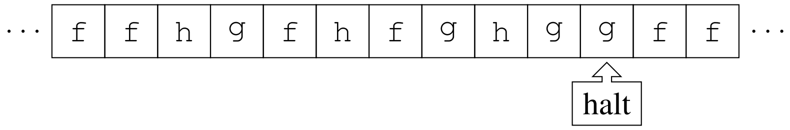
- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



· · ·	f	f	h	g	h	h	h	g	h	g	g	f	f	· · ·

the final configuration should be



· · ·	f	f	h	g	f	h	f	g	h	g	g	f	f	· · ·

You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhgghhgh`"

Question sheet 60

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M visits every state at most ten times when executed on input `00000`;

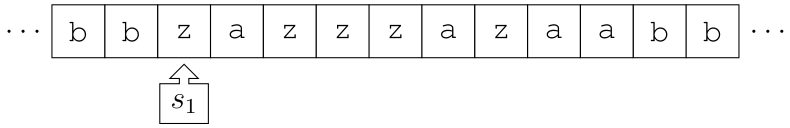
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

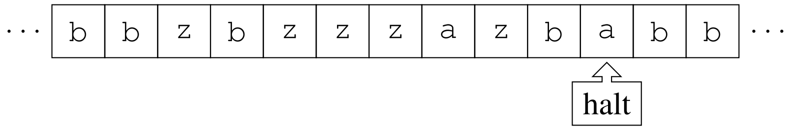
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Question sheet 61

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with more states than alphabet symbols;

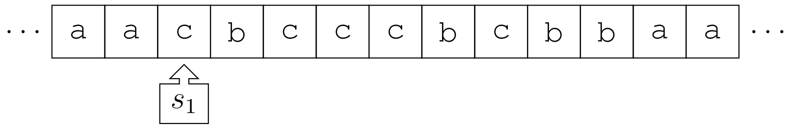
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

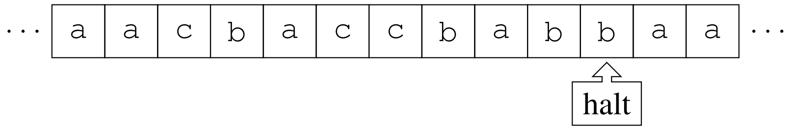
- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	a	a	c	b	c	c	c	b	c	b	b	a	a	···

the final configuration should be



···	a	a	c	b	a	c	c	b	a	b	b	a	a	···

You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bcbbccbbbccc`"

Question sheet 62

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

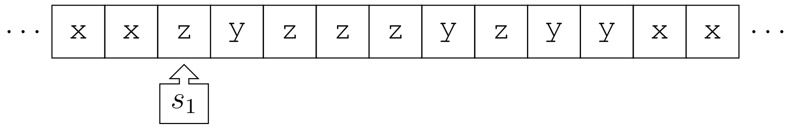
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

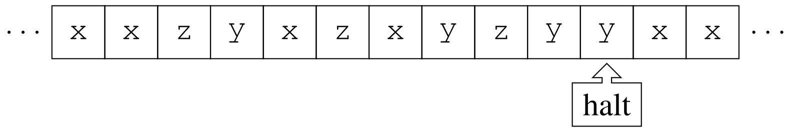
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyzyyzz`"

Question sheet 63

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M halts after more than 100 steps when executed on an empty tape;

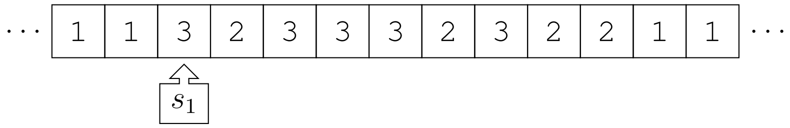
1.3) M recognizes Turing machines with less states than alphabet symbols.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

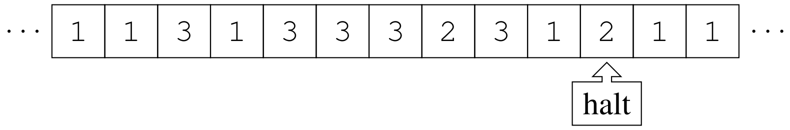
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	1	3	3	3	2	3	1	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333223323`"

Question sheet 64

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M never visits any state more than ten times when executed on an empty tape;

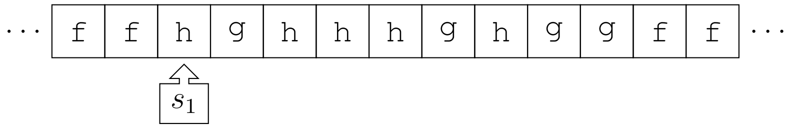
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

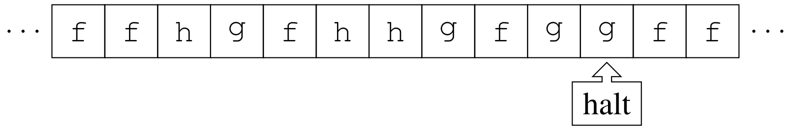
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 65

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M recognizes Turing machines with exactly 100 states;

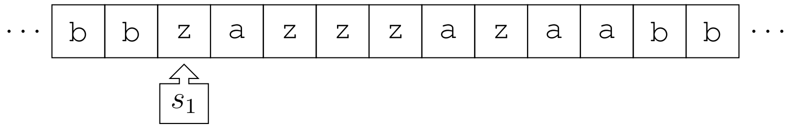
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

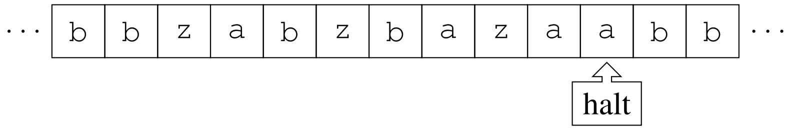
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 66

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more states than alphabet symbols;

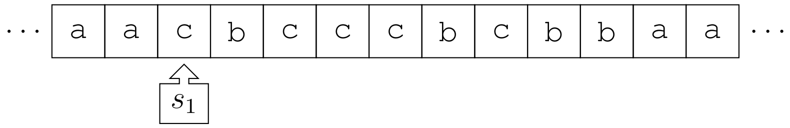
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

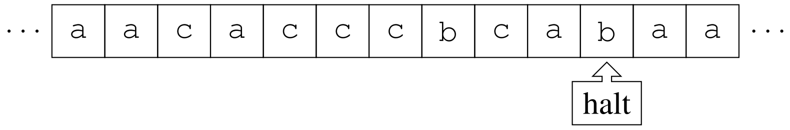
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbcbbcc`"

Question sheet 67

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M halts after more than 100 steps when executed on an empty tape;

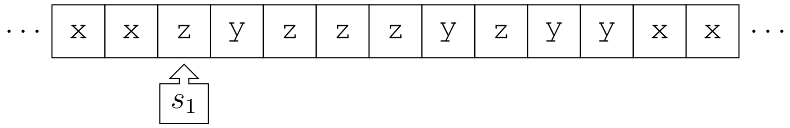
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

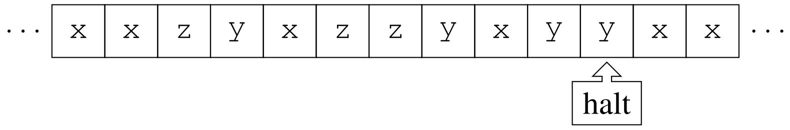
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyyzzyz`"

Question sheet 68

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on input `11111`;

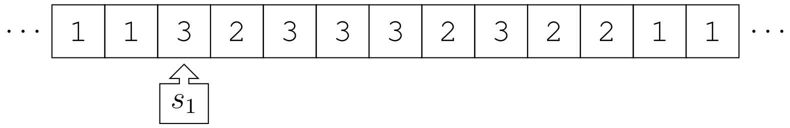
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

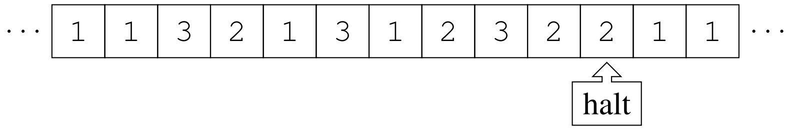
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`223322233323`"

Question sheet 69

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

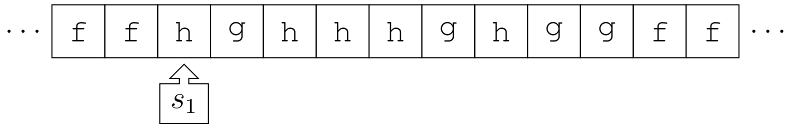
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

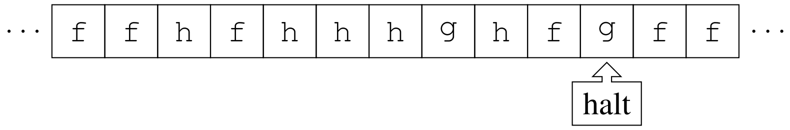
- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	f	f	h	g	h	h	h	g	h	g	g	f	f	···

the final configuration should be



···	f	f	h	f	h	h	h	g	h	f	g	f	f	···

You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ghgghhggghhh`"

Question sheet 70

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

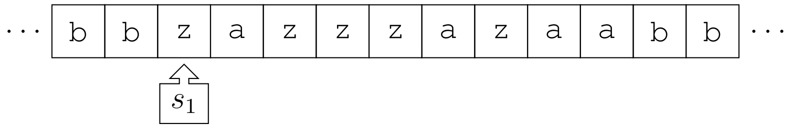
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

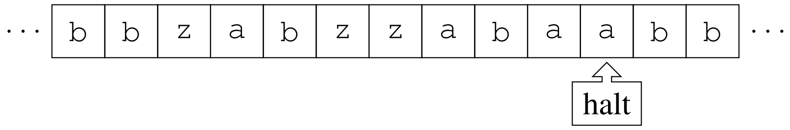
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzazaazz`"

Question sheet 71

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M halts after more than 100 steps when executed on an empty tape;

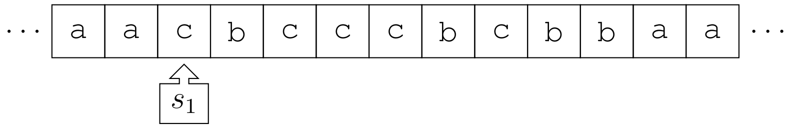
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

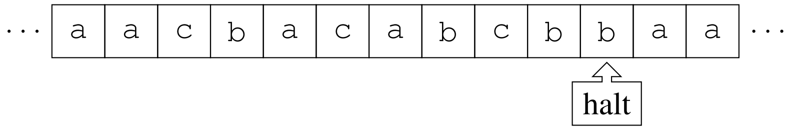
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 72

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

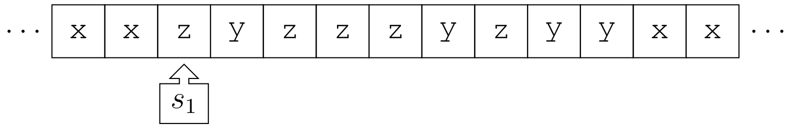
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

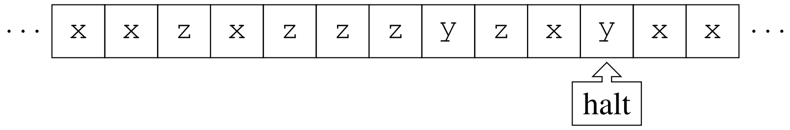
- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	x	x	z	y	z	z	z	y	z	y	y	x	x	⋯

the final configuration should be



⋯	x	x	z	x	z	z	z	y	z	x	y	x	x	⋯

You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yyzzyyyzzzyz`”

Question sheet 73

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

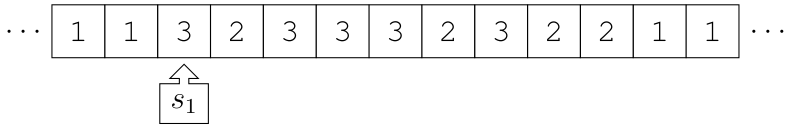
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

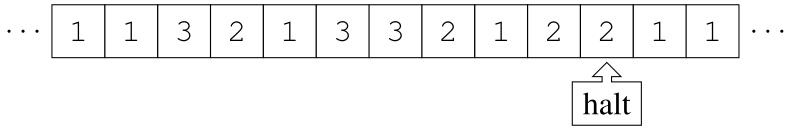
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	2	1	3	3	2	1	2	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 74

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with less than 100 states;

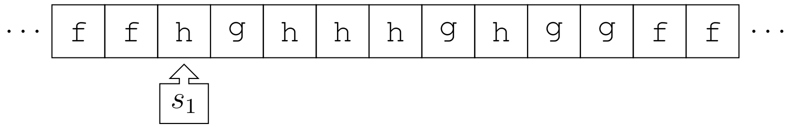
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

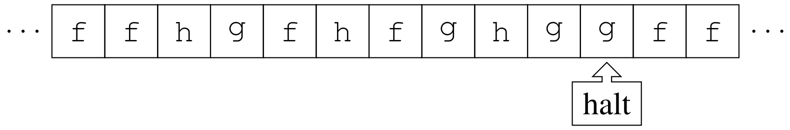
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhghgghh`"

Question sheet 75

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M halts after more than 100 steps when executed on an empty tape;

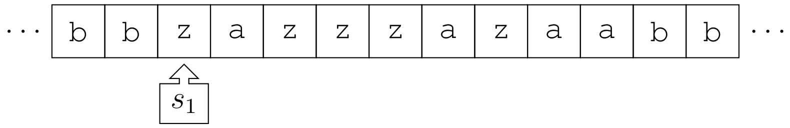
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

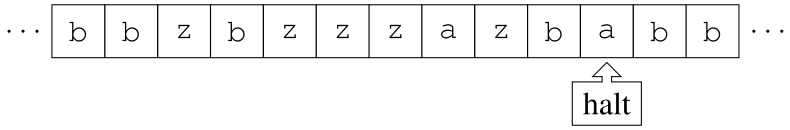
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 76

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

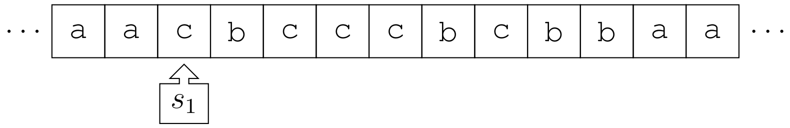
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

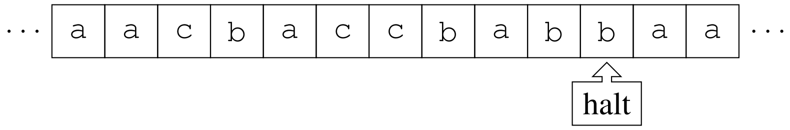
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbccbbbcccbc`"

Question sheet 77

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

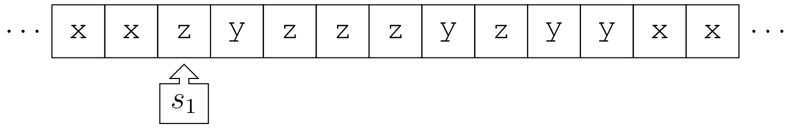
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

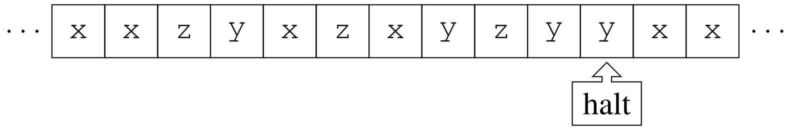
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{x}, \texttt{y}, \texttt{z}\}$, where ‘`x`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{y}, \texttt{z}\}$, surrounded by endless ‘`x`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol ‘`z`’ must be replaced with ‘`x`’ (i.e., the first ‘`z`’ must not be changed, the second must become ‘`x`’, the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`x`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yzyyzzyyyzzz`”

Question sheet 78

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M visits every state at most ten times when executed on input `00000`;

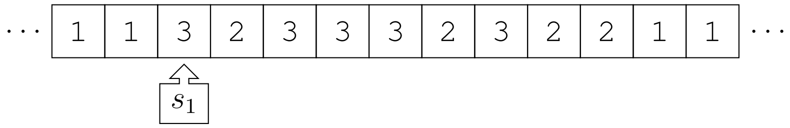
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

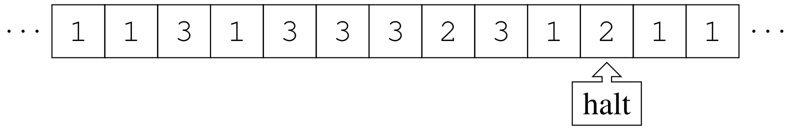
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	1	3	3	3	2	3	1	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333232233`"

Question sheet 79

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M never visits any state more than ten times when executed on an empty tape;

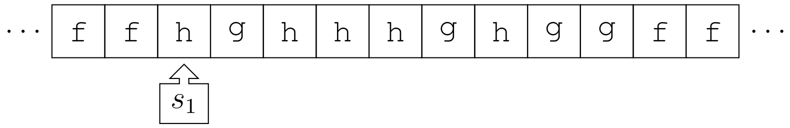
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

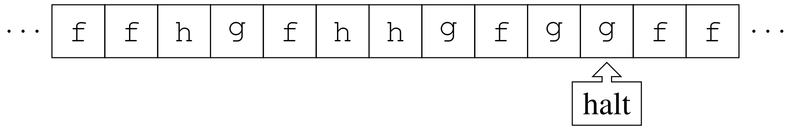
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`ggghhhgghhgh`”

Question sheet 80

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

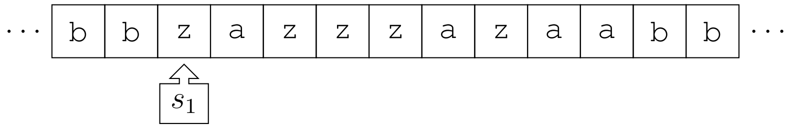
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

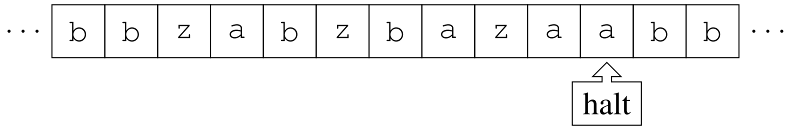
- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	b	b	z	a	z	z	z	a	z	a	a	b	b	···

the final configuration should be



···	b	b	z	a	b	z	b	a	z	a	a	b	b	···

You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Question sheet 81

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M visits every state at most ten times when executed on input `00000`;

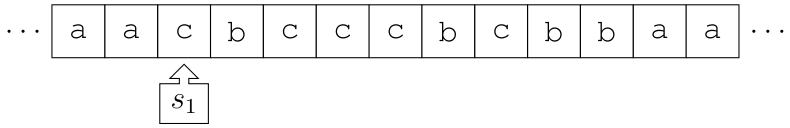
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

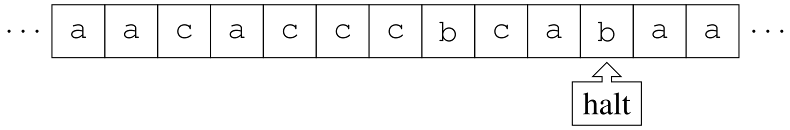
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bcbbccbbbccc`"

Question sheet 82

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

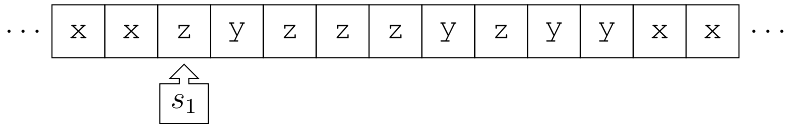
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

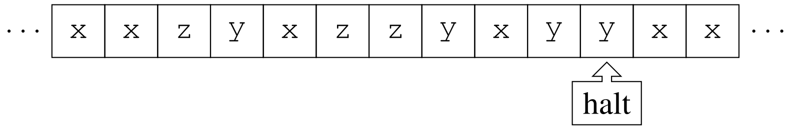
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyzyyzz`"

Question sheet 83

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

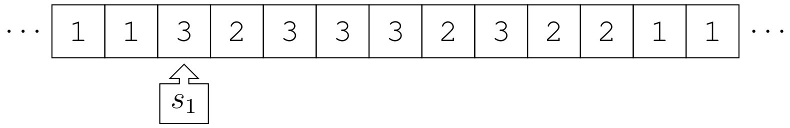
1.3) M recognizes Turing machines with less states than alphabet symbols.

Exercise 2

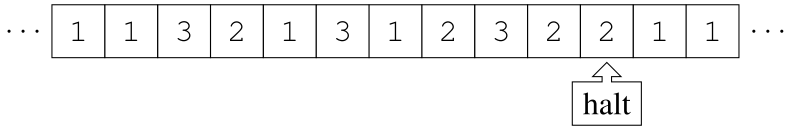
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333223323`"

Question sheet 84

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with less than 100 states;

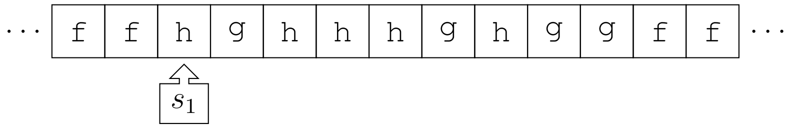
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

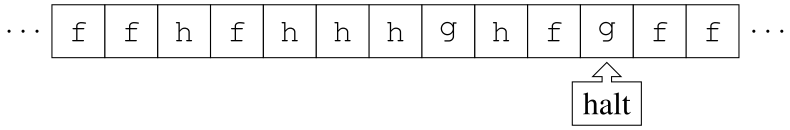
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 85

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M never visits any state more than ten times when executed on an empty tape;

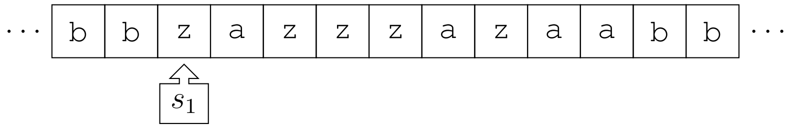
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

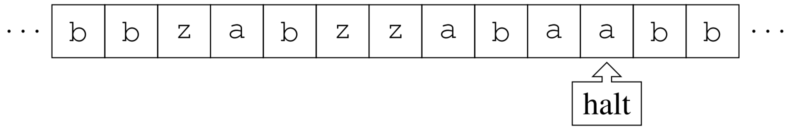
- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	b	b	z	a	z	z	z	a	z	a	a	b	b	···
			s_1											

the final configuration should be



···	b	b	z	a	b	z	z	a	b	a	a	b	b	···
											halt			

You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 86

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M recognizes Turing machines with more states than alphabet symbols;

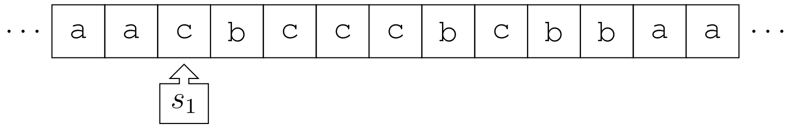
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

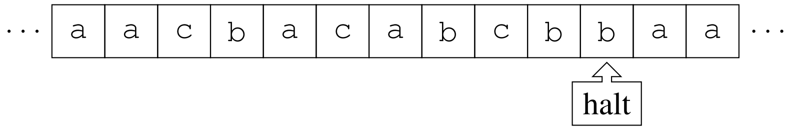
- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	a	a	c	b	c	c	c	b	c	b	b	a	a	⋯

the final configuration should be



⋯	a	a	c	b	a	c	a	b	c	b	b	a	a	⋯

You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbcbbcc`"

Question sheet 87

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M visits every state at most ten times when executed on input `00000`;

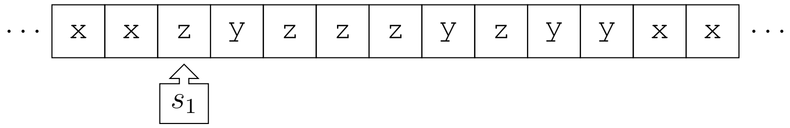
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

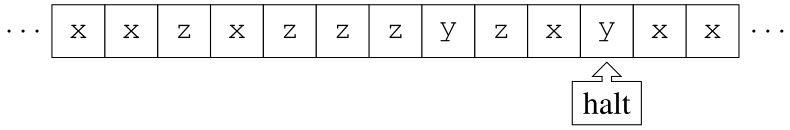
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyyzzyz`"

Question sheet 88

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with less states than alphabet symbols;

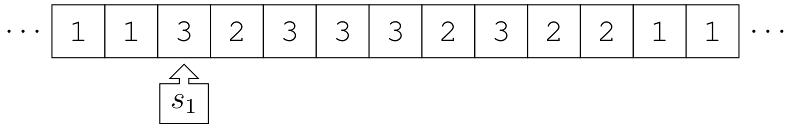
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

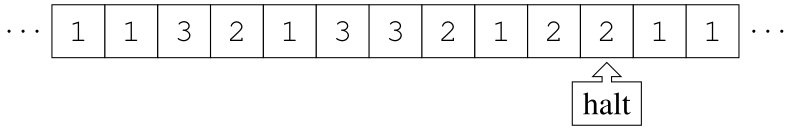
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`223322233323`"

Question sheet 89

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

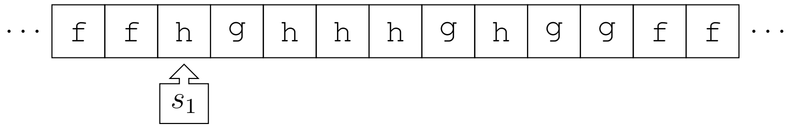
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

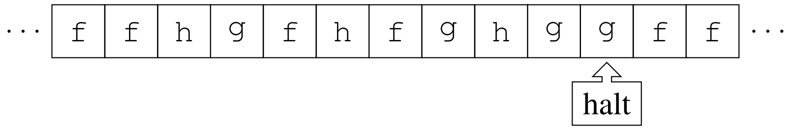
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ghgghhggghhh`"

Question sheet 90

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M visits every state at most ten times when executed on input `00000`;

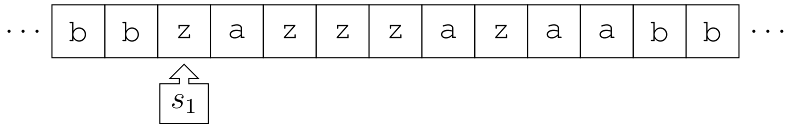
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

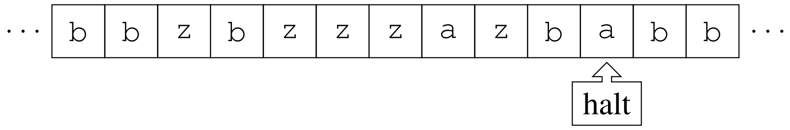
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzazaazz`"

Question sheet 91

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on an empty tape;

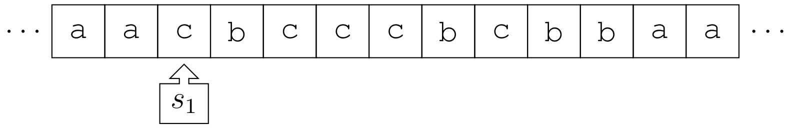
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

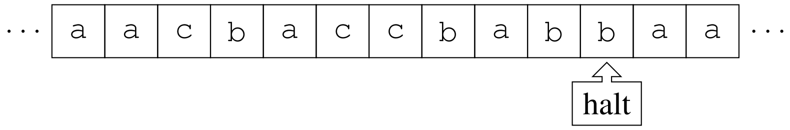
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 92

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

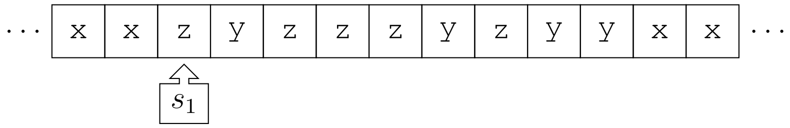
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

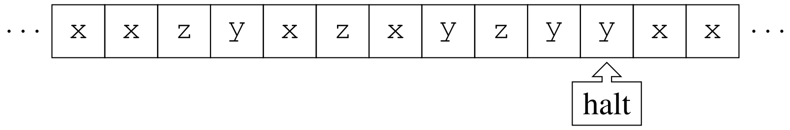
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyzzyyyzzzyz`"

Question sheet 93

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M visits every state at most ten times when executed on input `00000`;

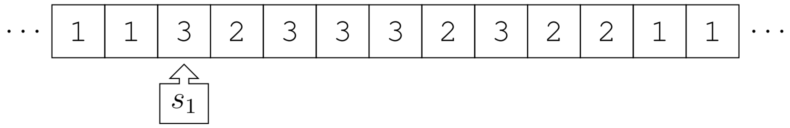
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

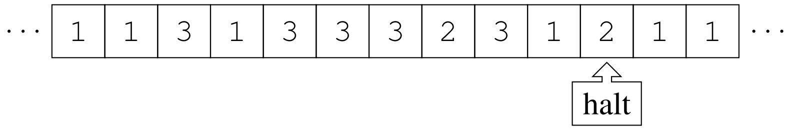
- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	1	3	3	3	2	3	1	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 94

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

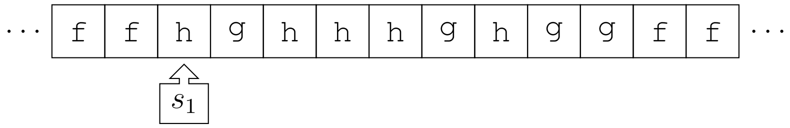
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

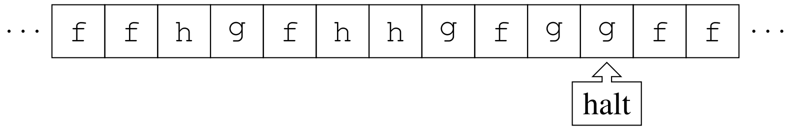
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhghgghh`"

Question sheet 95

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

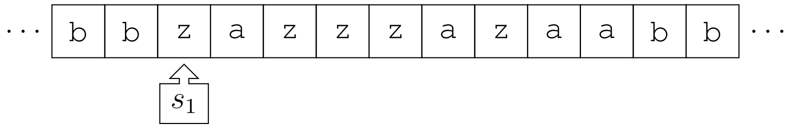
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

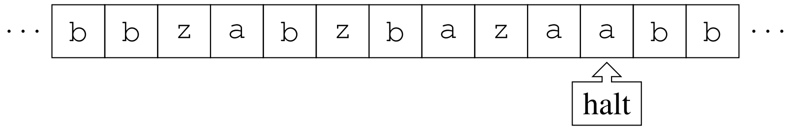
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 96

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M visits every state at most ten times when executed on input `00000`;

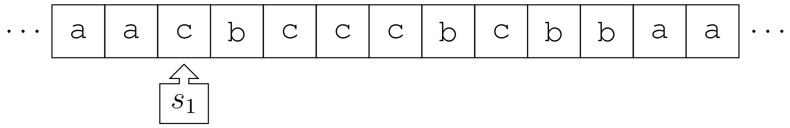
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

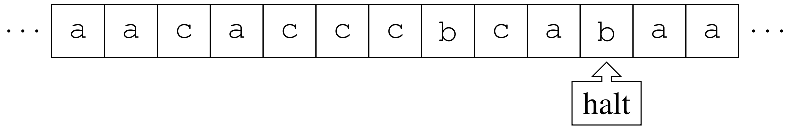
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbccbbbcccbc`"

Question sheet 97

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

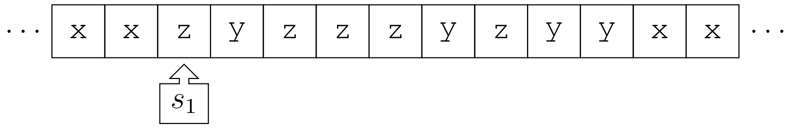
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

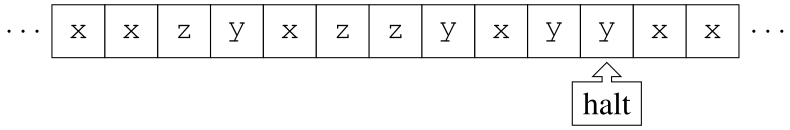
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where ‘`x`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless ‘`x`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every ‘`z`’ that immediately follows ‘`y`’ must be replaced with ‘`x`’ (i.e., every sequence ‘`yz`’ must become ’`yx`’).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`x`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yzyyzzyyyzzz`”

Question sheet 98

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

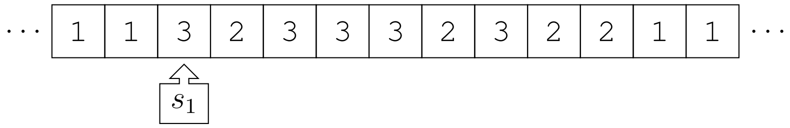
1.3) M recognizes Turing machines with less states than alphabet symbols.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

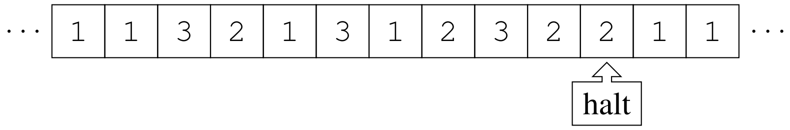
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	2	1	3	1	2	3	2	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333232233`"

Question sheet 99

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M halts after more than 100 steps when executed on an empty tape;

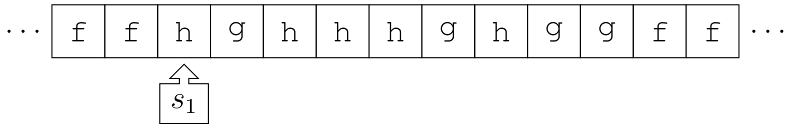
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

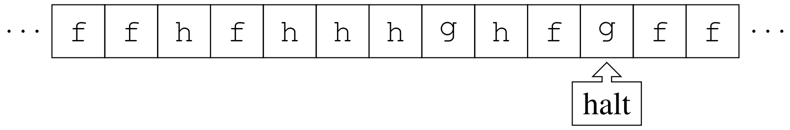
- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	f	f	h	g	h	h	h	g	h	g	g	f	f	···

the final configuration should be



···	f	f	h	f	h	h	h	g	h	f	g	f	f	···

You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhgghhgh`"

Question sheet 100

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

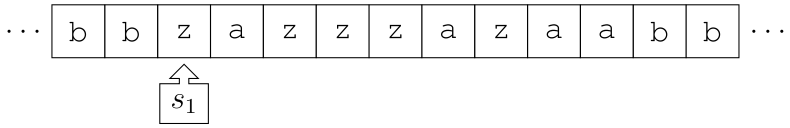
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

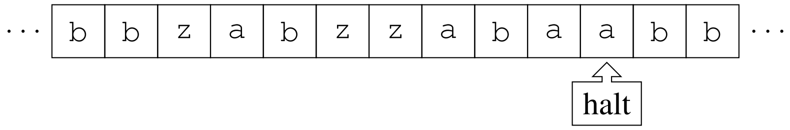
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Question sheet 101

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on input `11111`;

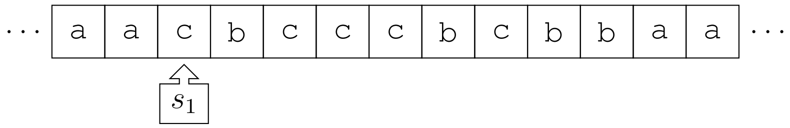
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

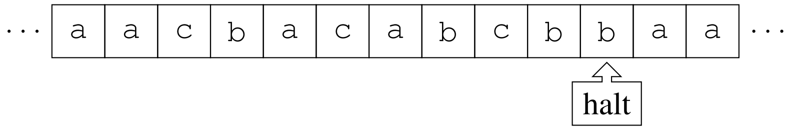
- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	a	a	c	b	c	c	c	b	c	b	b	a	a	⋯

the final configuration should be



⋯	a	a	c	b	a	c	a	b	c	b	b	a	a	⋯

You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bcbbccbbbccc`"

Question sheet 102

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

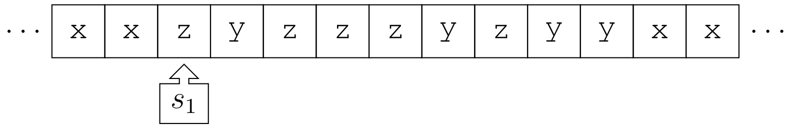
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

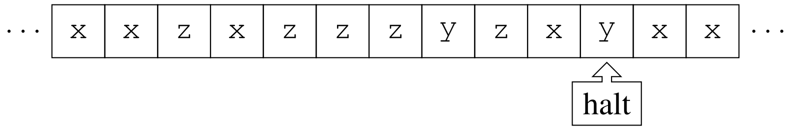
- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	x	x	z	y	z	z	z	y	z	y	y	x	x	···
			s_1											

the final configuration should be



···	x	x	z	x	z	z	z	y	z	x	y	x	x	···
											halt			

You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyzyyzz`"

Question sheet 103

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with less states than alphabet symbols;

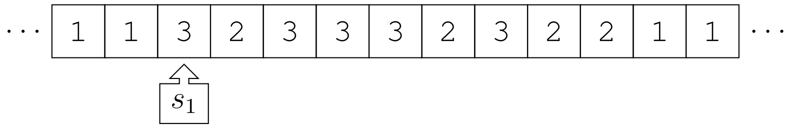
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

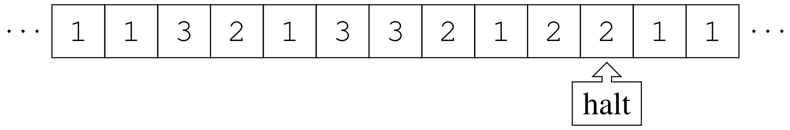
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333223323`"

Question sheet 104

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

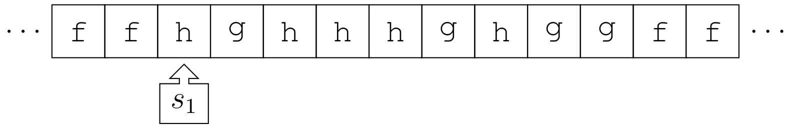
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

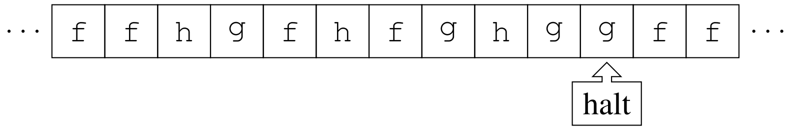
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 105

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

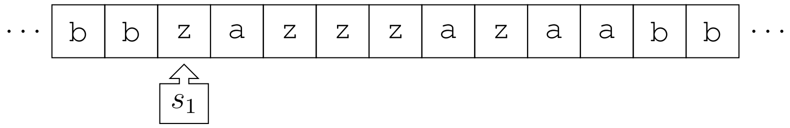
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

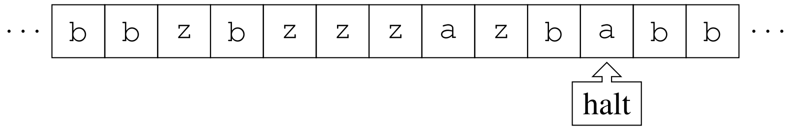
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 106

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more states than alphabet symbols;

1.2) M never visits any state more than ten times when executed on an empty tape;

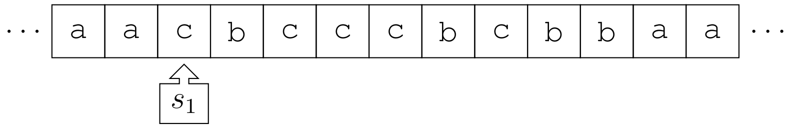
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

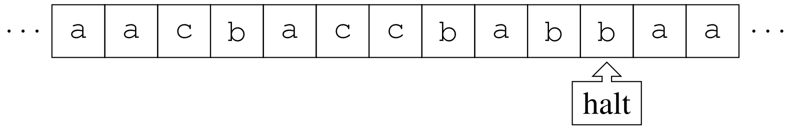
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbcbbcc`"

Question sheet 107

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M halts after more than 100 steps when executed on an empty tape;

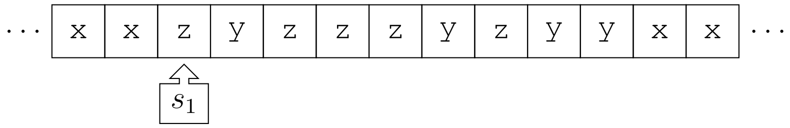
1.3) M recognizes Turing machines with more than 100 states.

Exercise 2

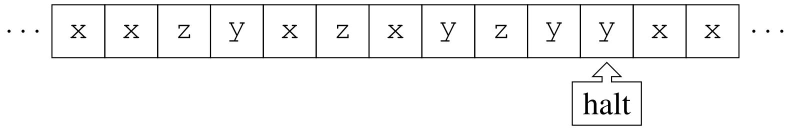
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yyyzzzyyzzyz`”

Question sheet 108

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with less states than alphabet symbols;

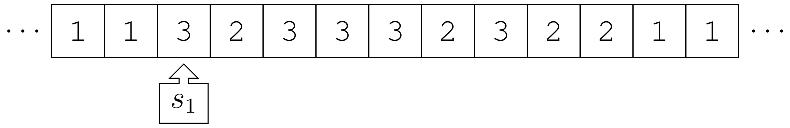
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

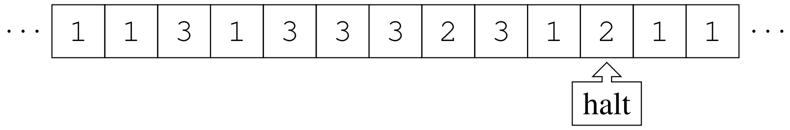
- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	1	3	3	3	2	3	1	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`223322233323`"

Question sheet 109

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

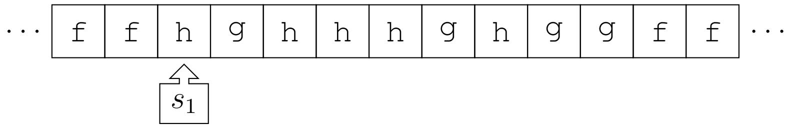
1.3) M recognizes Turing machines with less than 100 states.

Exercise 2

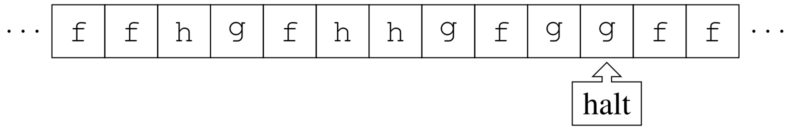
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ghgghhggghhh`"

Question sheet 110

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

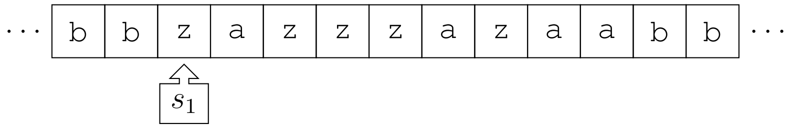
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

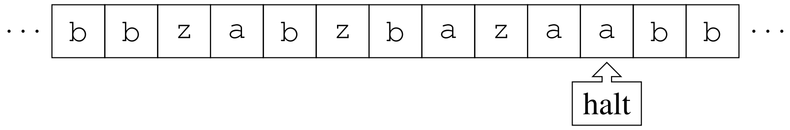
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzazaazz`"

Question sheet 111

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M halts after more than 100 steps when executed on an empty tape;

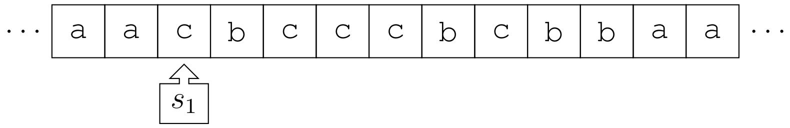
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

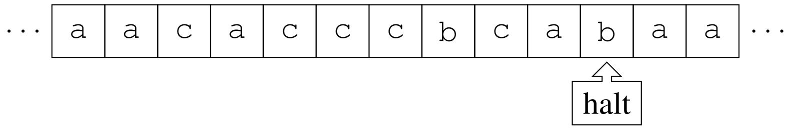
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{b}, \texttt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 112

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

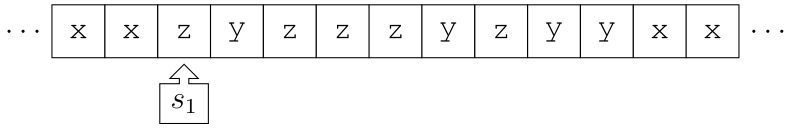
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

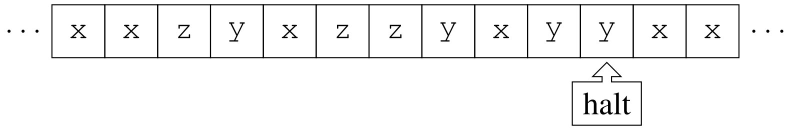
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyzzyyyzzzyz`"

Question sheet 113

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

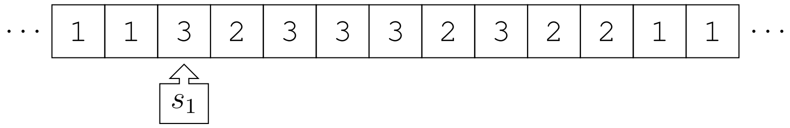
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

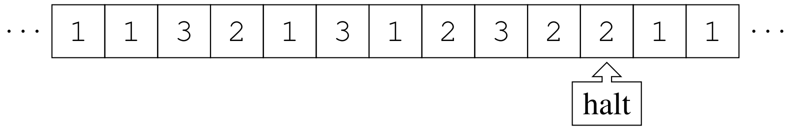
- the alphabet is $\Sigma = \{\mathtt{1}, \mathtt{2}, \mathtt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{2}, \mathtt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···
			s_1											

the final configuration should be



···	1	1	3	2	1	3	1	2	3	2	2	1	1	···
											halt			

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 114

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with less than 100 states;

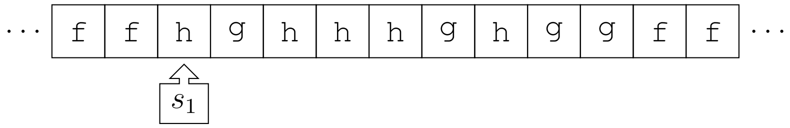
1.3) M visits every state at most ten times when executed on input `00000`.

Exercise 2

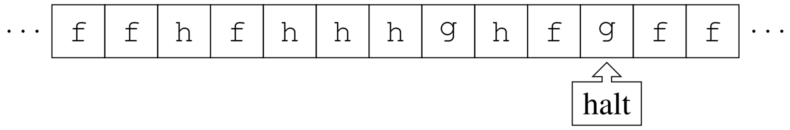
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{f}, \texttt{g}, \texttt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{g}, \texttt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhghgghh`"

Question sheet 115

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M recognizes Turing machines with exactly 100 states;

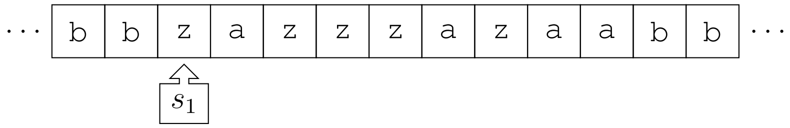
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

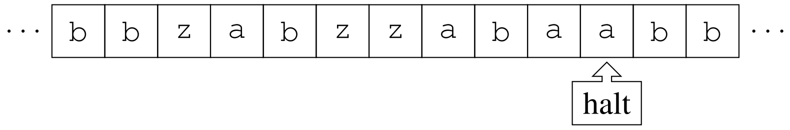
- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	b	b	z	a	z	z	z	a	z	a	a	b	b	···
			s_1											

the final configuration should be



···	b	b	z	a	b	z	z	a	b	a	a	b	b	···
											halt			

You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 116

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never enters the fifth state in its rule list when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on input `11111`;

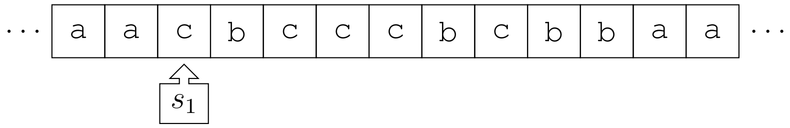
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

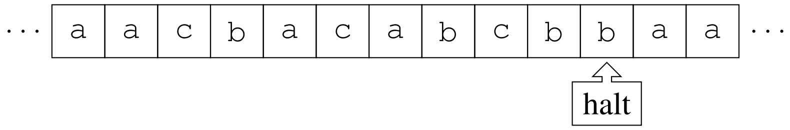
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbccbbbcccbc`"

Question sheet 117

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with more than 100 states;

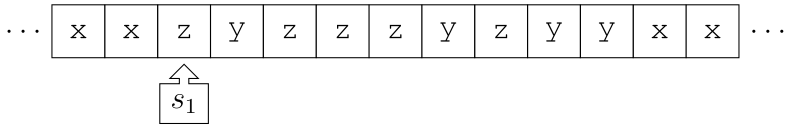
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

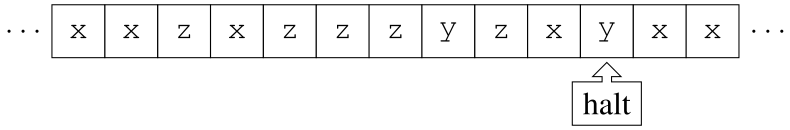
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{x}, \texttt{y}, \texttt{z}\}$, where ‘`x`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{y}, \texttt{z}\}$, surrounded by endless ‘`x`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol ‘`y`’ must be replaced with ‘`x`’ (i.e., the first ‘`y`’ must become ‘`x`’, the second must not be changed, the third must become ‘`x`’…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`x`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yzyyzzyyyzzz`”

Question sheet 118

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M recognizes Turing machines with less states than alphabet symbols;

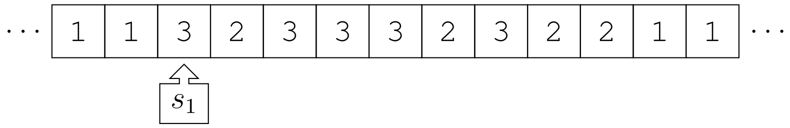
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

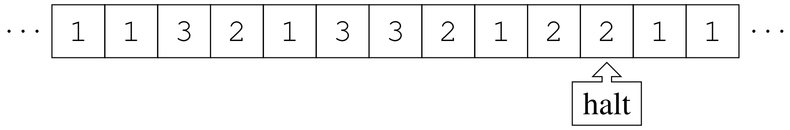
- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	1	1	3	2	3	3	3	2	3	2	2	1	1	⋯

the final configuration should be



⋯	1	1	3	2	1	3	3	2	1	2	2	1	1	⋯

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333232233`"

Question sheet 119

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

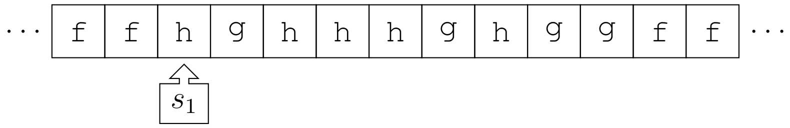
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

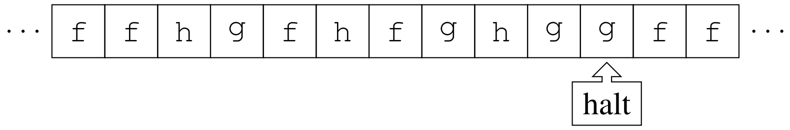
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhgghhgh`"

Question sheet 120

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

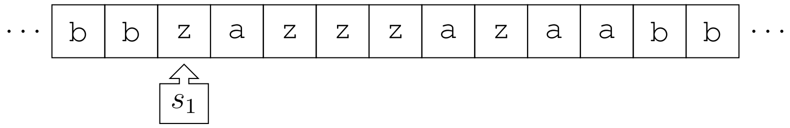
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

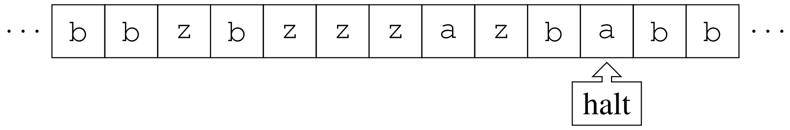
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Question sheet 121

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

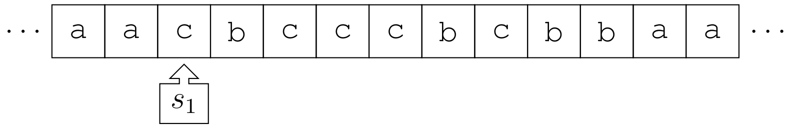
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

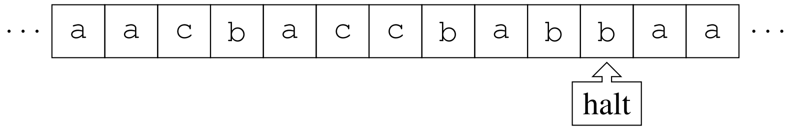
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bcbbccbbbccc`"

Question sheet 122

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M eventually enters the fifth state in its rule list when executed on an empty tape;

1.2) M recognizes Turing machines with more than 100 states;

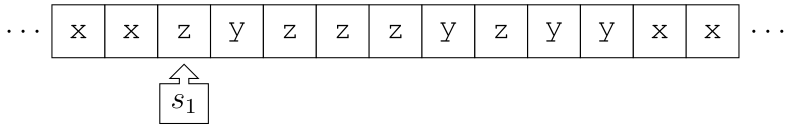
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

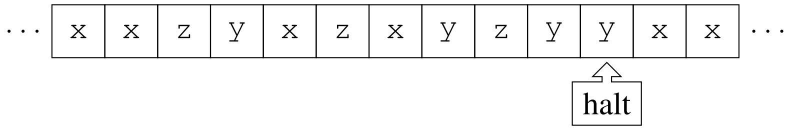
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`yyyzzzyzyyzz`”

Question sheet 123

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M visits every state at most ten times when executed on input `00000`;

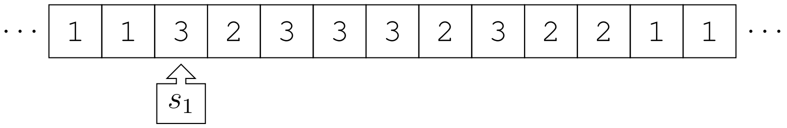
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

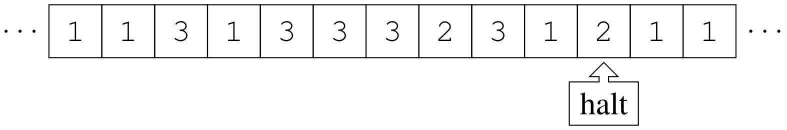
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where ‘`1`’ is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless ‘`1`’ symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol ‘`2`’ must be replaced with ‘`1`’ (i.e., the first ‘`2`’ must become ‘`1`’, the second must not be changed, the third must become ‘`1`’...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-‘`1`’ symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`222333223323`”

Question sheet 124

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

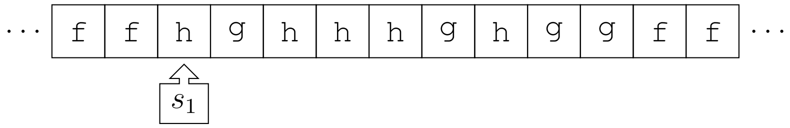
1.3) M recognizes Turing machines with less than 100 states.

Exercise 2

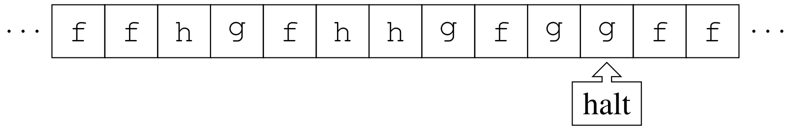
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`gghhggghhhgh`"

Question sheet 125

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M either performs less than 100 steps or runs forever when executed on an empty tape;

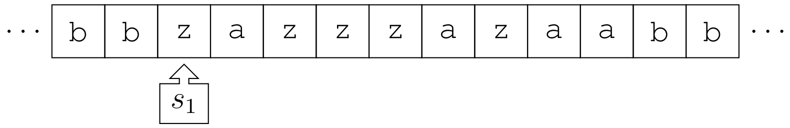
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

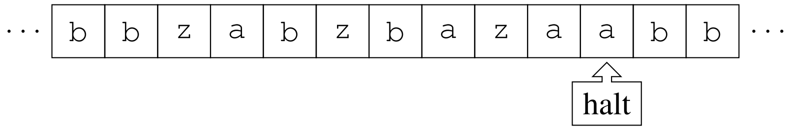
- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	b	b	z	a	z	z	z	a	z	a	a	b	b	···

the final configuration should be



···	b	b	z	a	b	z	b	a	z	a	a	b	b	···

You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`azaazzaaazzz`"

Question sheet 126

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with more states than alphabet symbols;

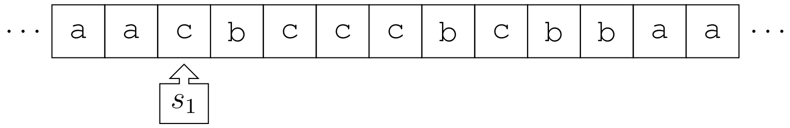
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

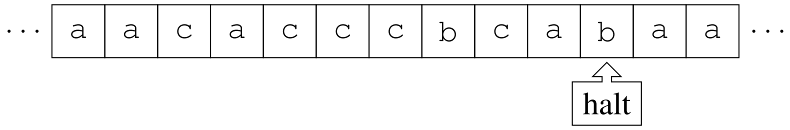
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`b`' must be replaced with '`a`' (i.e., the first '`b`' must become '`a`', the second must not be changed, the third must become '`a`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbcbbcc`"

Question sheet 127

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M never visits any state more than ten times when executed on an empty tape;

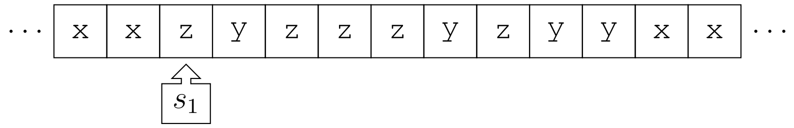
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

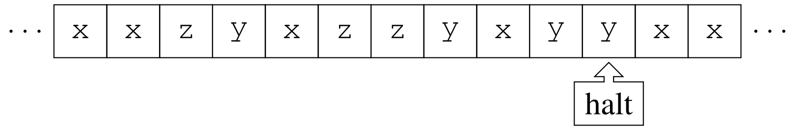
- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`y`' must be replaced with '`x`' (i.e., every sequence '`yz`' must become '`yx`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	x	x	z	y	z	z	z	y	z	y	y	x	x	···

the final configuration should be



···	x	x	z	y	x	z	z	y	x	y	y	x	x	···

You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyyzzzyyzzyz`"

Question sheet 128

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less states than alphabet symbols;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

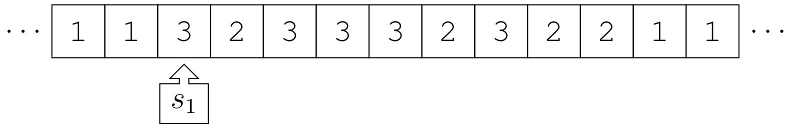
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

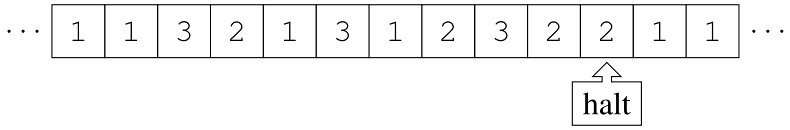
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`3`' must be replaced with '`1`' (i.e., the first '`3`' must not be changed, the second must become '`1`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

“`223322233323`”

Question sheet 129

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with less than 100 states;

1.2) M visits every state at most ten times when executed on input `00000`;

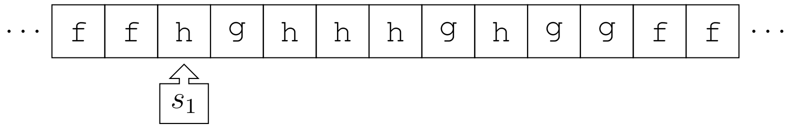
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

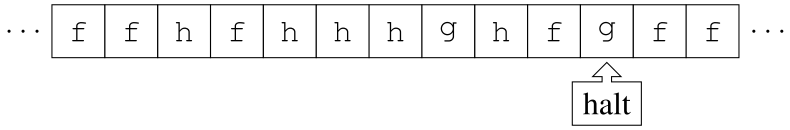
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`g`' must be replaced with '`f`' (i.e., the first '`g`' must become '`f`', the second must not be changed, the third must become '`f`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ghgghhggghhh`"

Question sheet 130

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with exactly 100 states;

1.2) M eventually enters the fifth state in its rule list when executed on an empty tape;

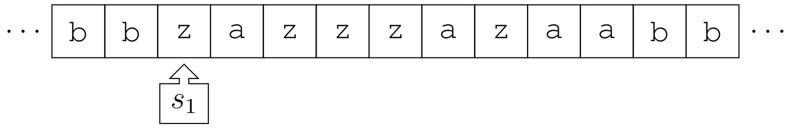
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

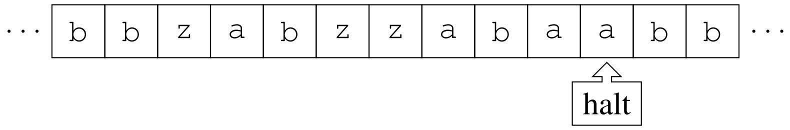
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`z`' that immediately follows '`a`' must be replaced with '`b`' (i.e., every sequence '`az`' must become '`ab`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzazaazz`"

Question sheet 131

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M recognizes Turing machines with more states than alphabet symbols;

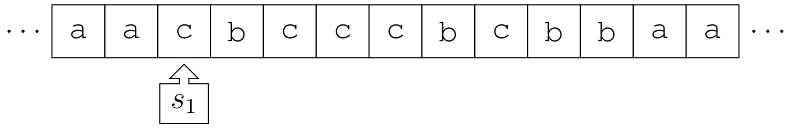
1.3) M never visits any state more than ten times when executed on input `11111`.

Exercise 2

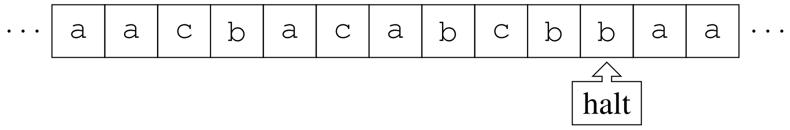
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{b}, \mathtt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbbcccbbccbc`"

Question sheet 132

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M visits every state at most ten times when executed on input `00000`;

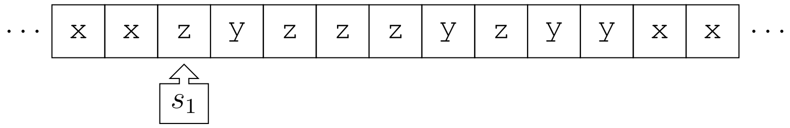
1.3) M never enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

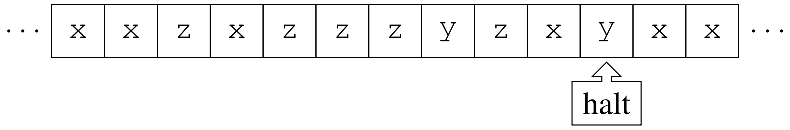
- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`y`' must be replaced with '`x`' (i.e., the first '`y`' must become '`x`', the second must not be changed, the third must become '`x`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	x	x	z	y	z	z	z	y	z	y	y	x	x	···

the final configuration should be



···	x	x	z	x	z	z	z	y	z	x	y	x	x	···

You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yyzzyyyzzzyz`"

Question sheet 133

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M either performs less than 100 steps or runs forever when executed on an empty tape;

1.2) M never visits any state more than ten times when executed on an empty tape;

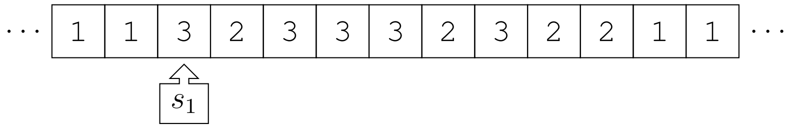
1.3) M recognizes Turing machines with less states than alphabet symbols.

Exercise 2

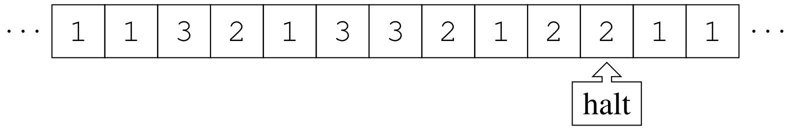
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`3`' that immediately follows '`2`' must be replaced with '`1`' (i.e., every sequence '`23`' must become '`21`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`232233222333`"

Question sheet 134

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M recognizes Turing machines with less than 100 states;

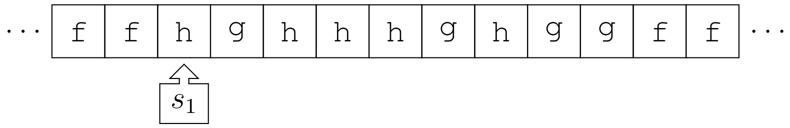
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

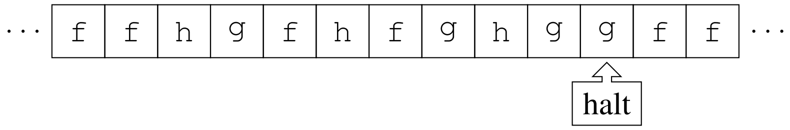
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`h`' must be replaced with '`f`' (i.e., the first '`h`' must not be changed, the second must become '`f`', the third must not be changed...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhghgghh`"

Question sheet 135

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with exactly 100 states;

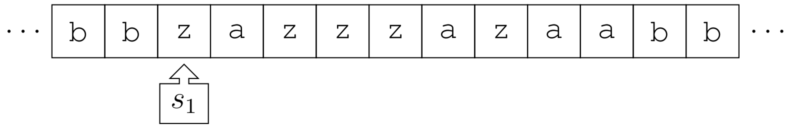
1.3) M halts after more than 100 steps when executed on an empty tape.

Exercise 2

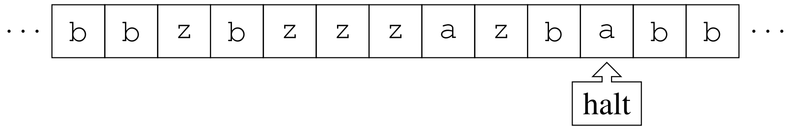
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\texttt{b}, \texttt{a}, \texttt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{a}, \texttt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`a`' must be replaced with '`b`' (i.e., the first '`a`' must become '`b`', the second must not be changed, the third must become '`b`'...).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aaazzzaazzaz`"

Question sheet 136

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on an empty tape;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

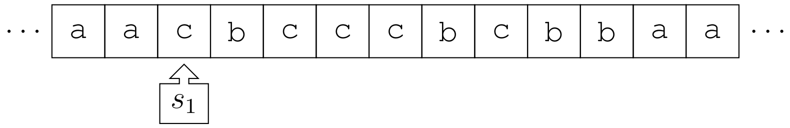
1.3) M recognizes Turing machines with more states than alphabet symbols.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

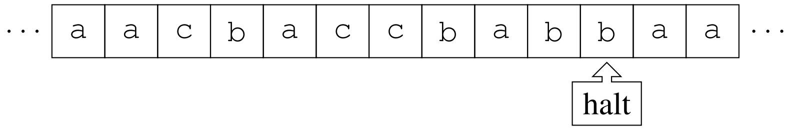
- the alphabet is $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$, where '`a`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{b}, \texttt{c}\}$, surrounded by endless '`a`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



⋯	a	a	c	b	c	c	c	b	c	b	b	a	a	⋯

the final configuration should be



⋯	a	a	c	b	a	c	c	b	a	b	b	a	a	⋯

You can assume that there is at least one non-'`a`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`bbccbbbcccbc`"

Question sheet 137

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M recognizes Turing machines with more than 100 states;

1.2) M never visits any state more than ten times when executed on input `11111`;

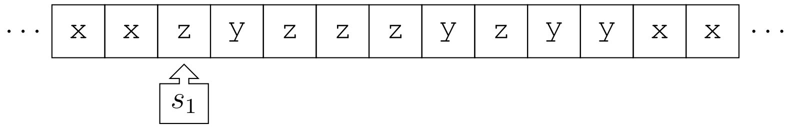
1.3) M either performs less than 100 steps or runs forever when executed on an empty tape.

Exercise 2

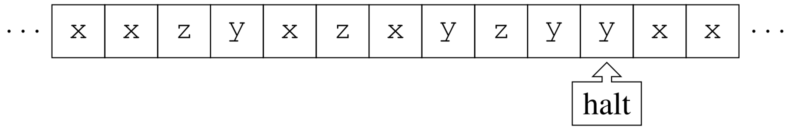
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, where '`x`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{y}, \mathtt{z}\}$, surrounded by endless '`x`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`x`' (i.e., the first '`z`' must not be changed, the second must become '`x`', the third must not be changed…).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`x`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`yzyyzzyyyzzz`"

Question sheet 138

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M visits every state at most ten times when executed on input `00000`;

1.2) M recognizes Turing machines with less states than alphabet symbols;

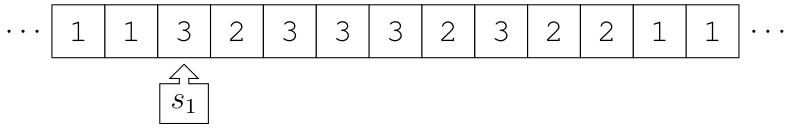
1.3) M eventually enters the fifth state in its rule list when executed on an empty tape.

Exercise 2

2.1) Write a Turing machine according to the following specifications:

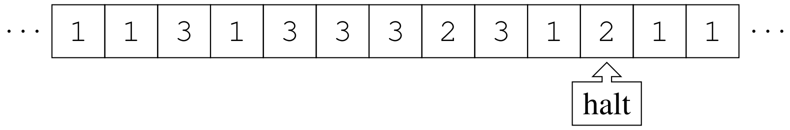
- the alphabet is $\Sigma = \{\texttt{1}, \texttt{2}, \texttt{3}\}$, where '`1`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\texttt{2}, \texttt{3}\}$, surrounded by endless '`1`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every odd occurrence of the symbol '`2`' must be replaced with '`1`' (i.e., the first '`2`' must become '`1`', the second must not be changed, the third must become '`1`'. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



···	1	1	3	2	3	3	3	2	3	2	2	1	1	···

the final configuration should be



···	1	1	3	1	3	3	3	2	3	1	2	1	1	···

You can assume that there is at least one non-'`1`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`222333232233`"

Question sheet 139

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M halts after more than 100 steps when executed on an empty tape;

1.2) M recognizes Turing machines with less than 100 states;

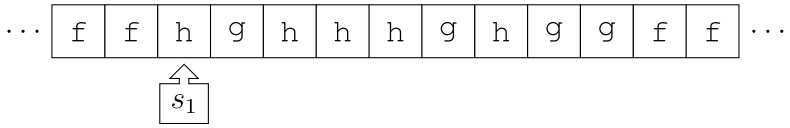
1.3) M never visits any state more than ten times when executed on an empty tape.

Exercise 2

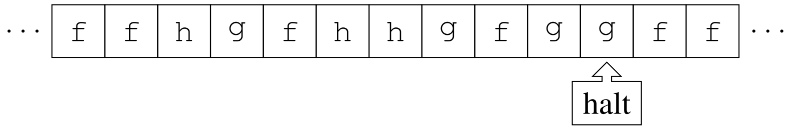
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}$, where '`f`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{g}, \mathtt{h}\}$, surrounded by endless '`f`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every '`h`' that immediately follows '`g`' must be replaced with '`f`' (i.e., every sequence '`gh`' must become '`gf`').
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`f`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`ggghhhgghhgh`"

Question sheet 140

Exercise 1

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

1.1) M never visits any state more than ten times when executed on input `11111`;

1.2) M never enters the fifth state in its rule list when executed on an empty tape;

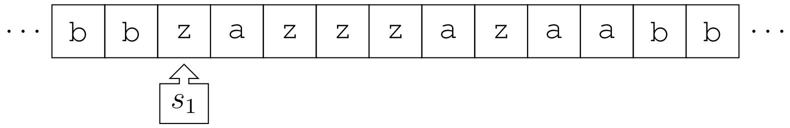
1.3) M recognizes Turing machines with exactly 100 states.

Exercise 2

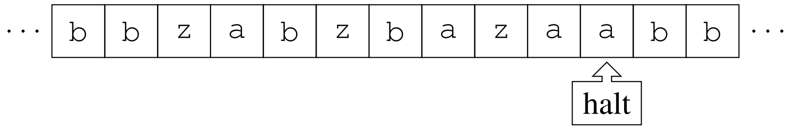
2.1) Write a Turing machine according to the following specifications:

- the alphabet is $\Sigma = \{\mathtt{b}, \mathtt{a}, \mathtt{z}\}$, where '`b`' is the default symbol;
- it has a single, bidirectional and unbounded tape;
- the input string is a finite sequence of symbols in $\{\mathtt{a}, \mathtt{z}\}$, surrounded by endless '`b`' symbols on both sides;
- the initial position of the machine is on the leftmost symbol of the input string;
- every even occurrence of the symbol '`z`' must be replaced with '`b`' (i.e., the first '`z`' must not be changed, the second must become '`b`', the third must not be changed. . .).
- the final position of the machine is at the righmost symbol of the output sequence.

For instance, in the following input case



the final configuration should be



You can assume that there is at least one non-'`b`' symbol on the tape, but considering the more general case in which the input might be the empty string is a bonus.

2.2) Show the sequence of steps that your machine performs on the input

"`aazzaaazzzaz`"

Answer guidelines to exercise 1

Every question sheet had a non-semantic recursive property (related to entering every state a bounded number of times), a non-semantic, non-recursive property (related in some way to the halting problem), and a semantic property (recognizing Turing machines) upon which Rice's Theorem could be invoked.

Non-semantic recursive properties

- M never visits any state more than ten times when executed on an empty tape
- M never visits any state more than ten times when executed on input `11111`
- M visits every state at most ten times when executed on input `00000`

In all cases, the property is clearly non-semantic (it is in no way related to the recognized language: different machines recognizing the same language may have it or not).

To decide whether M has that property, we can simulate it and keep a counter for each state; at every step, we increase the counter for the current state. As soon as a counter exceeds the limit of 10, we reject the machine. If the machine halts before we reject it, then we accept it. Since the number of states $|Q|$ is bounded, a decision will be reached within $10|Q|$ steps. Also, observe that one run is enough, because the property only cares about a specific input string.

Non-semantic, non-recursive properties

P_1 — M either performs less than 100 steps or runs forever when executed on an empty tape

P_2 — M eventually enters the fifth state in its rule list when executed on an empty tape

P_3 — M halts after more than 100 steps when executed on an empty tape

P_4 — M never enters the fifth state in its rule list when executed on an empty tape

In all cases, we can reduce the halting problem HALT′ to this property P: given M, we can always modify it in a simple way to transform the assertion "M eventually halts on an empty input" into the required property.

E.g., pick any machine N. Let N' be like N, but with 100 "do nothing" (or "back-and-forth") states at the beginning. Then HALT′$(N) = 1 - P_1(N') = P_3(N')$.

Or, let N' be like N with state number 5 renumbered to 6 and so forth; add a new fifth state with any rule, and replace all references to a halting state to a reference to the new state 5. Then HALT′$(N) = P_2(N') = 1 - P_4(N')$.

Semantic properties

- M recognizes Turing machines with more states than alphabet symbols
- M recognizes Turing machines with more than 100 states
- M recognizes Turing machines with less states than alphabet symbols

- M recognizes Turing machines with less than 100 states
- M recognizes Turing machines with exactly 100 states

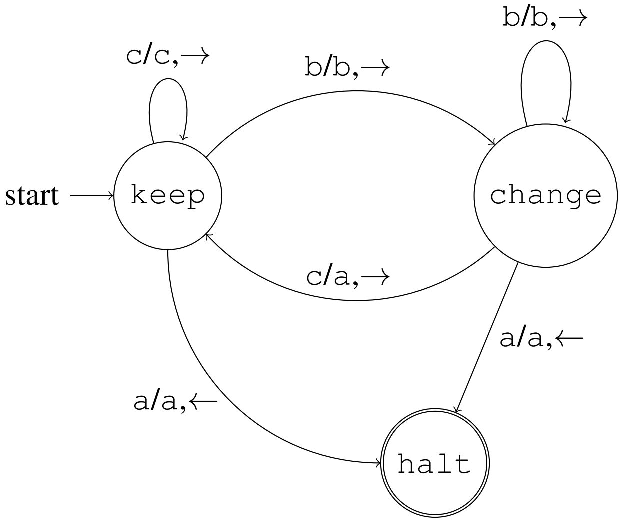
Observe that the property does not say "M has n states" (this would be a clearly computable non-semantic property, as counting states in a TM encoding is hardly a problem), but "M recognizes TMs that have n states"; i.e., we refer to the *language* (in this case a subset of TM encodings) that M is supposed to be able to recognize.

Answer guidelines to exercise 2

Let the alphabet be $\{\texttt{a}, \texttt{b}, \texttt{c}\}$; all exercises could be solved by a two-state machine (plus the halt state). Two possible representations of the Turing machine are shown; many other representations and transition rule sets are possible.

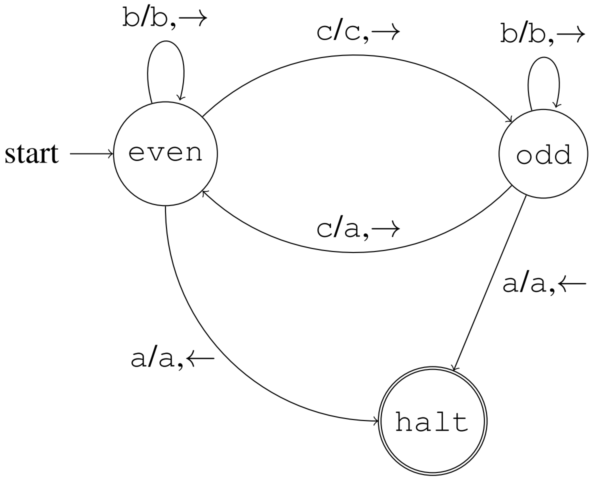
- every '`c`' that immediately follows '`b`' must be replaced with '`a`' (i.e., every sequence '`bc`' must become '`ba`').

	a	b	c
keep	a/←/halt	b/→/change	c/→/keep
change	a/←/halt	b/→/change	a/→/keep



- every even occurrence of the symbol '`c`' must be replaced with '`a`' (i.e., the first '`c`' must not be changed, the second must become '`a`', the third must not be changed...).

	a	b	c
even	a/←/halt	b/→/even	c/→/odd
odd	a/←/halt	b/→/odd	a/→/even



- every odd occurrence of the symbol ‘`b`’ must be replaced with ‘`a`’ (i.e., the first ‘`b`’ must become ‘`a`’, the second must not be changed, the third must become ‘`a`’. . .).

	a	b	c
even	a/←/halt	b/→/even	a/→/odd
odd	a/←/halt	b/→/odd	c/→/even